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*Original article*

# Bernoulli Differential Equations Solution Using Adomian Decomposition Method by Matlab

**Safaa Abu-Amr[1]*, Tebra Almajbri[2], Amina Mohammed[1]**

[1]*Department of Mathematics, Faculty of Science, University of Omar Al-Mokhtar, El-Baida, Libya*
[2]*Department of Materials Science and Engineering, Mathematics Major, Faculty of Engineering, University of Omar Al-Mokhtar, El-Baida, Libya*

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***Corresponding Email.*** *safaa.soliman@omu.edu.ly*

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**ABSTRACT**

*This paper uses the Adomoian decomposition method to solve Bernoulli differential equations, a type of nonlinear differential equation with numerous physical applications. The test problems included numerous Bernoulli differential equations with varied nonlinear component exponents, which were described using a decomposition-based numerical approach. The results are equally accurate in tables and graphs as the classical method.*

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## INTRODUCTION

Many issues can be represented as ordinary differential equations, particularly first-order Bernoulli differential equations, thus we must study and solve them. Bernoulli differential equations (BDE) are nonlinear equations named after J. Bernoulli, a Swiss scientist. They are equations characterized by their non-linearity and precise solutions. The equation has a non-linear term, which is a function of the independent variable elevated to a specific exponent, such as n. When $n = 0$ or $1$, the BDE is linear. Substitution is used to translate $n \geq 2$ into a linear form, allowing for linear solutions [1-4]. In this study, we use the Adomian decomposition method (ADM) to solve BDE with $n \geq 2$.

The (ADM) uses relation based on the Adomian polynomial to generate a solution for a series and since its presentation in the 1980s, the Adomian polynomial has undergone various modifications. The original Adomian polynomial is commonly utilized because to its simple and easy-to-memorize algorithm [5-8].

## METHODS

### Bernoulli differential equation $\frac{dy}{dx} + P(x)y = Q(x)y^n$

The Adomian Decomposition Method is extremely effective in solving nonlinear ordinary differential equations. Consider that the differential equation has the Bernoulli equation form.

$$\frac{dy}{dx} + P(x)y = Q(x)y^n \tag{1}$$

Where $P(x)$ and $Q(x)$ are arbitrary function of $x$, and $n$ is an arbitrary constant.
Assume the answer of Eq. (1) is given by the power series form.

$$y(x) = \sum_{n=0}^{\infty} y_n(x) \,. \tag{2}$$

The nonlinear term $y^n$ can be decomposed in terms of the Adomian polynomials $A_n(x)$, given by

$$y^n(x) = \sum_{n=0}^{\infty} A_n(x) \,. \tag{3}$$

Generally speaking, the Adomian polynomials are defined as follows for any function $f(t, x)$ [9]:

$$A_n = \frac{1}{n!}\frac{d^n}{d\epsilon^n} f\left(t, \sum_{i=0}^{\infty} \epsilon^i y_i\right)\Big|_{\epsilon=0}. \tag{4}$$

The first four Adomian polynomials are derived in the following form:

$$A_0 = f(t, y_0)\,, A_1 = y_1 f'(t, y_0)\,, A_2 = y_2 f'(t, y_0) + \frac{1}{2} y_1^2 f''(t, y_0)\,, \tag{5}$$

$$A_3 = y_3 f'(t, y_0) + y_1 y_2 f''(t, y_0) + \frac{1}{6} y_1^3 f'''(t, y_0)\ . \tag{6}$$

A few Adomian polynomials for the function $y^n$are [10]

$$A_0 = y_0^n, A_1 = n y_1 y_0^{n-1}, A_2 = n y_2 y_0^{n-1} + n(n-1)\frac{y_1^2}{2!}\ y_0^{n-2}, \tag{7}$$

$$A_3 = n y_3 y_0^{n-1} + n(n-1) y_1 y_2 y_0^{n-2} + n(n-1)(n-2)\frac{y_1^3}{3!}\ y_0^{n-3}\ . \tag{8}$$

Integrating Eq. (1) produces the integral equation.

$$y\,(x) = y\,(0) + \int_0^x [Q(x) y^n - P(x) y]\, dx, \tag{9}$$

Where (0) is the starting condition. The relationship can be obtained by replacing Eqs. (2) and (3) with Eq. (9)

$$\sum_{n=0}^{\infty} y_n(x) = y(0) + \int_0^x Q(x) \sum_{n=0}^{\infty} A_n(x)\, dx - \int_0^x P(x) \sum_{n=0}^{\infty} y_n(x)\, dx. \tag{10}$$

Eq. (10) is rewritten using the recursive forms.

$$y_0(x) = y\,(0)\,, \tag{11}$$

$$y_{k+1}(x) = \int_0^x [Q(x) A_k(x) - P(x) y_k(x)] dx. \tag{12}$$

From Eqs. (11) and (12), we obtain the semi-analytical solution of Eq. (1), given by

$$y(x) = \sum_{n=0}^{\infty} y_n(x). \tag{13}$$

## RESULTS

**Example 1:** Consider the Bernoulli differential equation

$$\frac{dy}{dx} = x^3 y^3 - xy\,, y(0) = 1$$

Become

$$\frac{dy}{dx} + xy = x^3 y^3, \tag{14}$$

With initial condition $y(0) = 1$, having the general solution

$$y\,(x) = \frac{1}{1+x^2}\,. \tag{15}$$

In this case $P\,(x) = x$ *and* $Q(x) = x^3$, respectively, and $n = 3$. Next, we compute a few Adomian polynomials for $y^3$,

$$A_0 = y_0^3, A_1 = 3 y_1 y_0^2, A_2 = 3 y_2 y_0^2 + 3 y_1^2\ y_0^1, A_3 = 3 y_3 y_0^2 + 6 y_1 y_2 y_0^1 + y_1^3 \tag{16}$$

Hence, we obtain

$$y_0(x) = y\,(0) = 1, \tag{17}$$

$$y_{k+1}(x) = \int_0^x [Q(x) A_k(x) - P(x) y_k(x)] dx. \tag{18}$$

Eq. (12) can be written recursively for $k = 0,1,2,3$ in the decomposed solutions

$$y_1(x) = \int_0^x [x^3 A_0(x) - x y_0(x)] dx = \frac{1}{4} x^4 - \frac{1}{2} x^2, \tag{19}$$

$$y_2(x) = \int_0^x [x^3 A_1(x) - x y_1(x)] dx = \frac{3}{32} x^8 - \frac{7}{24} x^6 + \frac{1}{8} x^4, \tag{20}$$

$$y_3(x) = \int_0^x [x^3 A_2(x) - x y_2(x)] dx = \frac{5}{128} x^{12} - \frac{11}{64} x^{10} + \frac{17}{96} x^8 - \frac{1}{48} x^6, \tag{21}$$

$$y_4(x) = \int_0^x [x^3 A_3(x) - x y_3(x)] dx = \frac{35}{2048} x^{16} - \frac{165}{1792} x^{14} + \frac{103}{768} x^{12} - \frac{37}{960} x^{10} - \frac{1}{384} x^8, \tag{22}$$

$$y\,(x) \approx y_0(x) + y_1(x) + y_2(x) + y_3(x) + y_4(x)$$
$$= \frac{35}{2048} x^{16} - \frac{165}{1792} x^{14} + \frac{133}{768} x^{12} - \frac{202}{960} x^{10} - \frac{103}{384} x^8 - \frac{15}{48} x^6 + \frac{3}{8} x^4 - \frac{1}{2} x^2 + 1. \tag{23}$$

On the other hand, from the exact solution (15) it is easy to obtain

$$y\,(x) = \frac{1}{1+x^2}$$
$$= \frac{35}{2048} x^{16} - \frac{165}{1792} x^{14} + \frac{133}{768} x^{12} - \frac{202}{960} x^{10} - \frac{103}{384} x^8 - \frac{15}{48} x^6 + \frac{3}{8} x^4 - \frac{1}{2} x^2 + 1 \tag{24}$$

Clearly again, the solution (23) obtained by the (ADM) is identical to the exact solution (24).

***Table 1. Computed Approximate and Exact Solution for example 1***

| x | Exact | Approximate | Abs.Error |
|---|---|---|---|
| **0.1** | 9.9009900E-01 | 9.9018361E-01 | 8.5448405E-05 |
| **0.2** | 9.6153846E-01 | 9.6162323E-01 | 8.8161095E-05 |
| **0.3** | 9.1743119E-01 | 9.1692095E-01 | 5.5616326E-04 |
| **0.4** | 8.6206896E-01 | 8.6020490E-01 | 2.1623067E-03 |
| **0.5** | 8.0000000E-01 | 7.9679416E-01 | 4.0072935E-03 |
| **0.6** | 7.3529411E-01 | 7.3262555E-01 | 3.6292387E-03 |
| **0.7** | 6.7114093E-01 | 6.7338836E-01 | 3.3486627E-03 |
| **0.8** | 6.0975609E-01 | 6.2334517E-01 | 2.2286089E-02 |
| **0.9** | 15.524861E-01 | 5.8244315E-01 | 5.4222116E-02 |

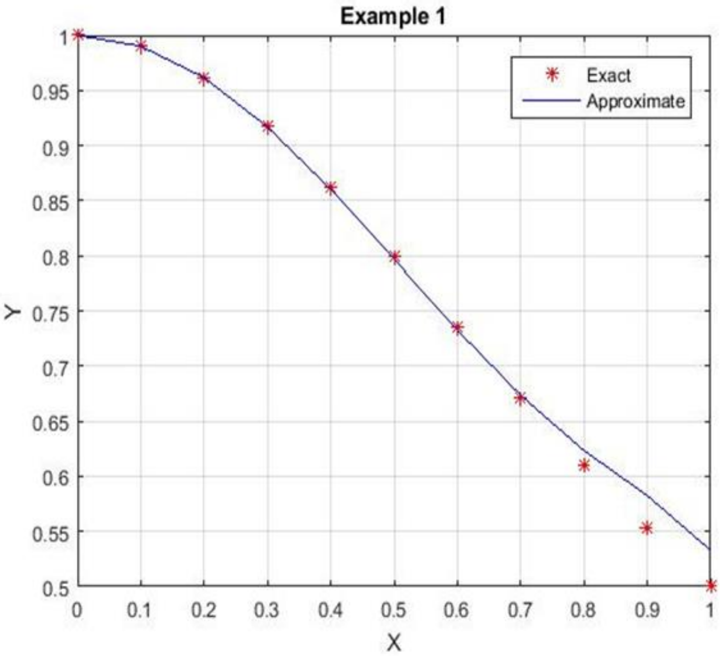
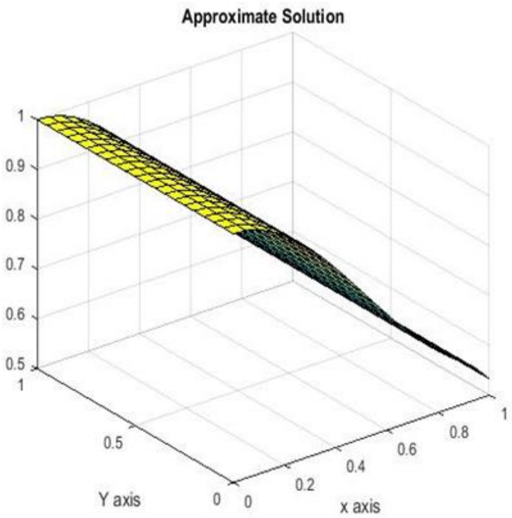



***Figure 1. Numerical results for Example 1.***



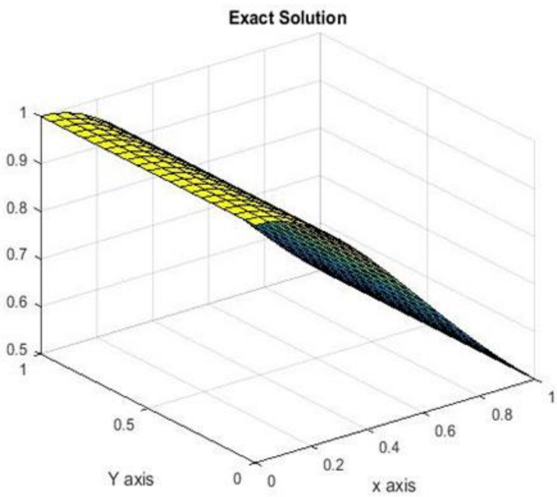




***Figure 2. Approximate Solution*** ***Figure 3. Exact Solution***

**Example 2:** Consider the Bernoulli differential equation

$$y' = 2xy + 2x^3y^2 , y(0) = 1$$

Become

$$y' - 2xy = 2x^3y^2, \quad (25)$$

With $y(0) = 1$, having the general solution

$$y\,(x) = \frac{1}{(1-x^2)}. \quad (26)$$

In this case $P\,(x) = -2x$ *and* $Q(x) = 2x^3$, respectively, and $n = 2$. Next we compute a few Adomian polynomials for $y^2$,

$$A_0 = y_0^3 = y_0 = y(0) = 1, A_1 = 2y_1y_0, A_2 = 2y_2y_0 + y_1^2 , A_3 = 2y_3y_0 + 2y_1y_2 \quad (27)$$

Hence, we obtain

$$y_0(x) = y\,(0) = 1, \quad (28)$$

$$y_{k+1}(x) = \int_0^x [Q(x)A_k(x) - P(x)y_k(x)]dx. \quad (29)$$

Eq. (12) can be written recursively for $k = 0, 1, 2, 3$ in the decomposed solutions

$$y_1(x) = \int_0^x [2x^3A_0(x) + 2xy_0(x)]dx = \frac{1}{2}x^4 + x^2, \quad (30)$$

$$y_2(x) = \int_0^x [2x^3A_1(x) + 2xy_1(x)]dx = \frac{1}{4}x^8 + \frac{5}{6}x^6 + \frac{1}{2}x^4, \quad (31)$$

$$y_3(x) = \int_0^x [2x^3A_2(x) + 2xy_2(x)]dx = \frac{1}{8}x^{12} + \frac{7}{12}x^{10} + \frac{17}{24}x^8 + \frac{1}{6}x^6, \quad (32)$$

$$y_4(x) = \int_0^x [2x^3A_3(x) + 2xy_3(x)]dx = \frac{1}{8}x^{16} + \frac{19}{56}x^{14} + \frac{25}{36}x^{12} + \frac{49}{120}x^{10} + \frac{1}{24}x^8, \quad (33)$$

$$y\,(x) \approx y_0(x) + y_1(x) + y_2(x) + y_3(x) + y_4(x)$$

$$= \frac{1}{8}x^{16} + \frac{19}{56}x^{14} + \frac{59}{72}x^{12} + \frac{119}{120}x^{10} + x^8 + x^6 + x^4 + x^2 + 1. \quad (34)$$

On the other hand, from the exact solution (26) it is easy to obtain

$$y\,(x) = \frac{1}{(1-x^2)} = \frac{1}{8}x^{16} + \frac{19}{56}x^{14} + \frac{59}{72}x^{12} + \frac{119}{120}x^{10} + x^8 + x^6 + x^4 + x^2 + 1 \quad . \quad (35)$$

Clearly again, the solution (34) obtained by the (ADE) is identical to the exact solution (35).

***Table 2. Computed approximate and exact solution for example 2.***

| x | Exact | Approximate | Abs.Error |
|---|---|---|---|
| **0.1** | 1.0101010101 | 1.0101010100 | 1.0107514825E-12 |
| **0.2** | 1.0416666666 | 1.0416666649 | 1.6388618462E-09 |
| **0.3** | 1.0989010989 | 1.0989009179 | 1.6467015050E-07 |
| **0.4** | 1.1904761904 | 1.1904700562 | 5.1527639801E-06 |
| **0.5** | 1.3333333333 | 1.3332223498 | 8.3237602597E-05 |
| **0.6** | 1.5625000000 | 1.5611332835 | 8.7469849616E-04 |
| **0.7** | 1.9607843137 | 1.9474678303 | 6.7914065014E-03 |
| **0.8** | 2.7777777777 | 2.6607477317 | 4.2130816583E-02 |
| **0.9** | 5.2631578947 | 4.1059967520 | 2.1986061710E-01 |

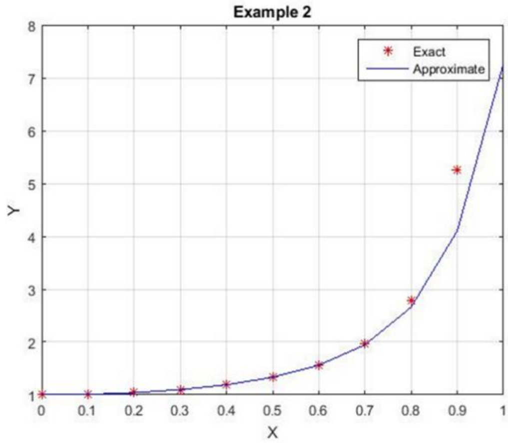



***Figure 4. Numerical results for example 2.***

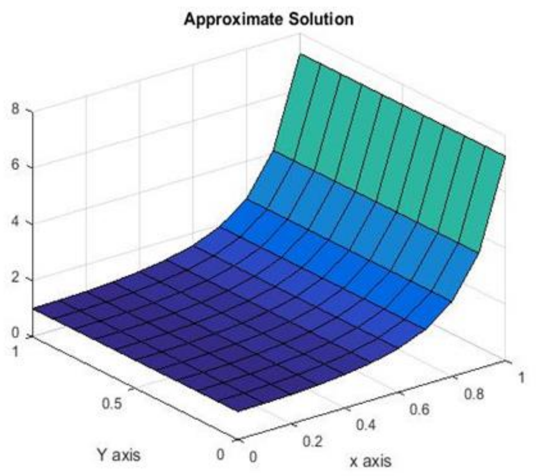



*Figure 5. Approximate Solution*

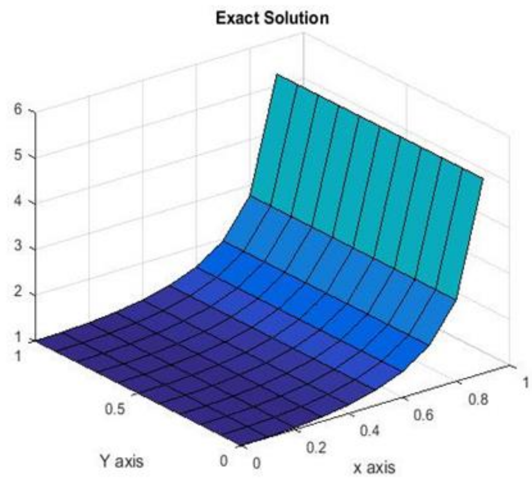



*Figure 6. Exact Solution*

## CONCLUSION

In this study, we succeeded in applying the (ADM) to (BDE). We used it to solve actual problems, which were considered to have a positive index of the nonlinear term. The Adomian decomposition method gave results similar to the analytical solutions of the Bernoulli differential equation. The figures clearly show the exact solutions when compared to those found using the (ADM).

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# حل معادلات برنولي التفاضلية باستخدام طريقة التحليل الأدومي باستخدام ماتلاب

**صفاء أبو عمرو[1]*، تبرا المجبري[2]، أمينة محمد[1]**

[1]قسم الرياضيات، كلية العلوم، جامعة عمر المختار، البيضاء، ليبيا
[2]قسم علوم وهندسة المواد، تخصص الرياضيات، كلية الهندسة، جامعة عمر المختار، البيضاء، ليبيا

**المستخلص**
تستخدم هذه الورقة طريقة التحلل الأدوموي لحل معادلات برنولي التفاضلية، وهو نوع من المعادلات التفاضلية غير الخطية ذات التطبيقات الفيزيائية العديدة. تضمنت مشاكل الاختبار العديد من معادلات برنولي التفاضلية ذات الأسس المكونة غير الخطية المتنوعة، والتي تم وصفها باستخدام نهج عددي قائم على التحلل. النتائج دقيقة بنفس القدر في الجداول والرسوم البيانية مثل الطريقة الكلاسيكية.
**الكلمات المفتاحية**. طريقة التحليل الأدومي، معادلة برنولي التفاضلية، الحلول التحليلية والتقريبية.