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Original article

Probabilistic Analysis of Different Redundant Complex System with Reper for the Unites

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ABSTRACT

This paper presents the reliability and MTTF analysis of a two-state complex with repairable system, consisting of two sub-systems A and two sub-systems B arranged in series, incorporating the concept of hardware failures. Laplace transforms of the various state probabilities have been derived and then reliability of the complex system, at any time t, has been computed by inversion process. MTTF has also been evaluated; availability and steady-state availability for system are derived. The failure times of operating units and repair time of failed units are exponential distributed. Certain important results have been evaluated as special cases. Also, few graphical illustrations are also given at the end to high-light the important results.

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INTRODUCTION

Earlier researchers [1-3] have studied the reliability and MTTF for various complex equipment's, keeping in view the concept of human and hardware failure [1-3]. Previous study reported the reliability and MTTF analysis of non-repairable parallel redundant complex system under hardware and human failures [4]. Other researchers had studied the human error and partial hardware failure modeling of parallel and standby redundant system [5]. Also, previous researchers had studied the stochastic analysis of a compound redundant system involving human failure as a matter-of-fact human failure is defined as a failure to perform a prescribed task which could result in damage to the equipment and property [6].

There exist a number of causes for human error; e.g., lack of good job environments, poor training or skill of the operating personnel and so on. On the other hand, hardware failure occurs due to flaws in design, poor quality control, poor maintenance, etc. This type of study can be found in reference. In this paper; the authors have considered a repairable complex system consisting of two subsystems A and B. The subsystem A has a two-unit active parallel system whereas the subsystem B hast unit alone. The two subsystems are arranged in series. Both the units of subsystem A suffer two types of failure viz; hardware and human whereas subsystem B suffers only one type of failure. With the aid of Laplace transforms of the various state probabilities have been derived and then reliability is obtained by inversion process. Moreover, an important parameter of reliability, i.e., MTTF (mean time to failure), system availability and steady-state availability are derived. The failure times of operating units and repair time of failed units are exponential distributed. The effects of additional repair in this system performance are shown in tables and graphically.

This paper presents the mean time to system failure, pointwise availability of the system at time t and steady state availability and point wise reliability of the system at time t and steady state availability**.** In this system the following assumptions and notations are used to analyze the system. Initially, the system is in good state, the system has two states, viz; good and failed, a failed unit can be repaired, Hardware failures for all the units are also constant, Failures are statistically independent, two units connected in parallel redundancy suffer two types of failures, namely constant hardware failure and in the complex system, only one change can take place in the state of the system at any time.

Notations

$P_i(t)$	probability that the system is in state S_i at any time t, for i=0, 1,2,…, 10
s	Laplace-transform variable,
$F(s)$	Laplace-transform of $F(t)$,
λ_A	the constant hardware failure rate of a unit of sub-system A,
λ_B	the constant hardware failure rate of sub-system B,
μ_A	the constant repair rate from hardware failure of a unit for the sub-system A,
μ_B	the constant repair rate of sub-system B.

α the constant hardware failure rate of a unit of sub-system A when the second unit has already failed,

β the constant hardware failure rate of a unit of sub-system B when the second unit has already failed,

Stochastic behavior of the system

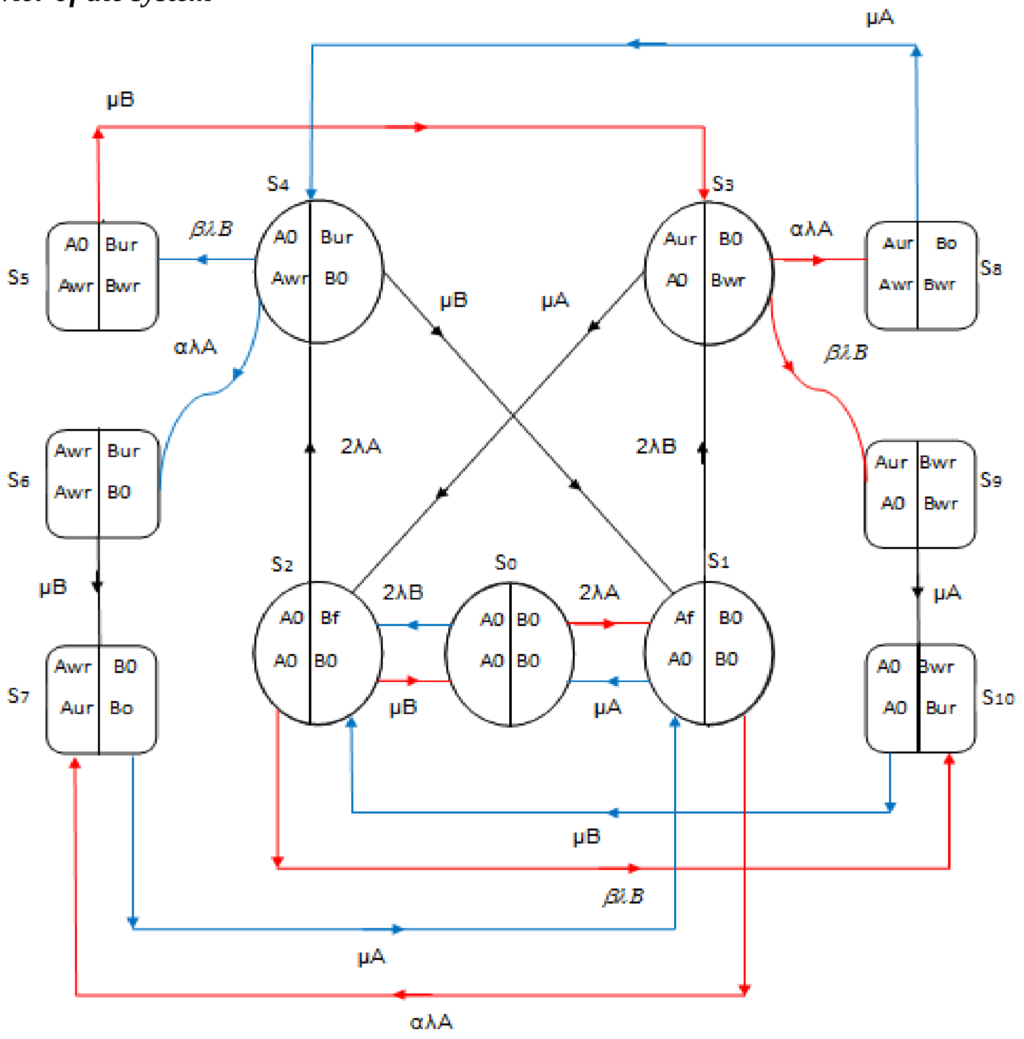
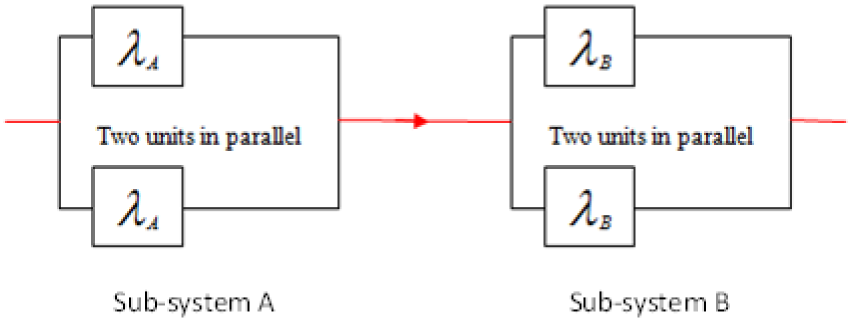


Figure 1. The states of the system



where, O = unit in the operative mode,

F = unit in the total hardware failure mode,

Cor UR = unit is under repair **g states:-**

$S_0 = \begin{pmatrix} \\ A_O, B_O \end{pmatrix}, S_1 = \begin{pmatrix} \\ A_O, B_O \end{pmatrix}, S_2 = \begin{pmatrix} \\ A_O, B_O \end{pmatrix}, S_3 = \begin{pmatrix} \\ A_O, B_{WF} \end{pmatrix}, S_4 = \begin{pmatrix} A_O, B_{Ur} \\ A_{wr}, B_O \end{pmatrix},$

$S_5 = \begin{pmatrix} A_O, B_{ur} \\ A_{wr}, B_{wr} \end{pmatrix}, S_6 = \begin{pmatrix} A_{wr}, B_{ur} \\ A_{wr}, B_O \end{pmatrix}, S_7 = \begin{pmatrix} A_{wr}, B_O \\ A_{ur}, B_O \end{pmatrix}, S_8 = \begin{pmatrix} A_{ur}, B_O \\ A_{wr}, B_{wr} \end{pmatrix}, S_9 = \begin{pmatrix} A_{ur}, B_{wr} \\ A_O, B_{wr} \end{pmatrix},$

$S_{10} = \begin{pmatrix} A_O, B_{wr} \\ A_O, B_{ur} \end{pmatrix}$

SYSTEM RELIABILITY

The system reliability $R(t)$ is the probability of failure-free operation of the system in (0, t]. To derive an expression for the reliability of the system, we restrict the transitions of the Markov process to the up states, viz. S_0, S_1,and S_2, S_3, S_4.Using the figure (1); we derive the following differential equations:

$$\frac{dp_0(t)}{dt} + (2\lambda_A + 2\lambda_B)p_0(t) = \mu_A p_1(t) + \mu_B p_2(t),$$
$$\frac{dp_1(t)}{d(t)} + (\alpha\lambda_A + 2\lambda_B + \mu_A)p_1(t) = 2\lambda_A p_0(t) + \mu_B \, p_4(t),$$
$$\frac{dP_2(t)}{dt} + (2\lambda_A + \beta\lambda_B + \mu_B)P_2(t) = 2\lambda_B p_0(t) + \mu_A P_3(t),$$
$$\frac{dp_3(t)}{dt} + (\mu_A + \beta\lambda_B + \alpha\lambda_A)p_3(t) = 2\lambda_B p_1(t),$$
$$\frac{dp_4(t)}{d(t)} + (\beta\lambda_B + \alpha\lambda_A + \mu_B)p_4(t) = 2\lambda_A p_2(t),$$
(1-5)

Using initial conditions, $P_0(0) = 1, P_i(0) = 0$, where $i = 1, 2, 3, 4$. Taking Laplace transforms, of the equations (1-5), we get

$$P_0(s)=\frac{-4\lambda_a\lambda_b\lambda_a\lambda_b+(s+x_1)(s+x_3)(s+x_2)(s+x_4)}{\prod_{r=1}^{5}(s-s_r)},$$

$$P_1(s)=\frac{(s+x_1)(4\lambda_a\lambda_b\mu_b+2\lambda_a(s+x_2)(s+x_4))}{\prod_{r=1}^{5}(s-s_r)},$$

$$P_2(s)=\frac{(4\lambda_a\lambda_b\mu_a+2\lambda_b(s+x_1)(s+x_3)(s+x_2))}{\prod_{r=1}^{5}(s-s_r)},$$

$$P_3(s)=\frac{2\lambda_b(-4\lambda_a\lambda_b\mu_b-2\lambda_a(s+x_2)(s+x_4))}{\prod_{r=1}^{5}(s-s_r)},$$

$$P_4(s)=\frac{-2\lambda_a(-4\lambda_a\lambda_b\mu_a-2\lambda_b(s+x_1)(s+x_3))}{\prod_{r=1}^{5}(s-s_r)}, \qquad (6\text{-}10)$$

Now taking inverse Laplace transforms of equations(6-10), we get

$$P_0(t)=\sum_{i=1}^{5}\frac{-4\lambda_a\lambda_b\lambda_a\lambda_b+(s_i+x_1)(s_i+x_3)(s_i+x_2)(s_i+x_4)}{\prod_{r=1,r\neq i}^{5}(s_i-s_r)}e^{s_i t},$$

$$P_1(t)=\sum_{i=1}^{5}\frac{(s_i+x_1)(4\lambda_a\lambda_b\mu_b+2\lambda_a(s_i+x_2)(s_i+x_4))}{\prod_{r=1,r\neq i}^{5}(s_i-s_r)}e^{s_i t},$$

$$P_2(t)=\sum_{i=1}^{5}\frac{(4\lambda_a\lambda_b\mu_a+2\lambda_b(s_i+x_1)(s_i+x_3)(s_i+x_2))}{\prod_{r=1,r\neq i}^{5}(s_i-s_r)}e^{s_i t},$$

$$P_3(t)=\sum_{i=1}^{5}\frac{2\lambda_b(-4\lambda_a\lambda_b\mu_b-2\lambda_b(s_i+x_2)(s_i+x_4))}{\prod_{r=1,r\neq i}^{5}(s_i-s_r)}e^{s_i t},$$

$$P_4(t)=\sum_{i=1}^{5}\frac{-2\lambda_a(-4\lambda_a\lambda_b\mu_a-2\lambda_b(s_i+x_1)(s_i+x_3))}{\prod_{r=1,r\neq i}^{5}(s_i-s_r)}e^{s_i t}, \qquad (11\text{-}15)$$

were, $x_1=(\alpha\lambda_A+\beta\lambda_B+\mu_A)$, $x_2=(\alpha\lambda_A+\beta\lambda_B+\mu_B)$, $x_3=(\alpha\lambda_A+2\lambda_B+\mu_A)$ and. $x_4=(2\lambda_A+\beta\lambda_B+\mu_B)$,

Then the system reliability is given by

$$R(t)=p_0(t)+p_1(t)+p_2(t)+p_3(t)+p_4(t),$$

$$=\sum_{i=1}^{5}[4\lambda_a\lambda_b(2\lambda_a\mu_a+2\lambda_b\mu_b-\mu_a\mu_b+B_i(\mu_a+D_i))+A_i\,2\lambda_a(2\lambda_b\mu_b+B_iD_i)$$

$$+C_i(4\lambda_a\lambda_b+B_i(2\lambda_b+D_i))/\prod_{r=1,r\neq i}^{5}(s_i-s_r)]*e^{s_i t} \qquad (12)$$

where, $A_i=(S_i+x_1), B_i=(S_i+x_2), C_i=(S_i+x_3), D_i=(S_i+x_4)$

s_1, s_2 and s_3, s_4 are roots of the polynomial of the expand the determinant for the following matrix:

$$\begin{bmatrix} (S+2\lambda_a+2\lambda_b) & \mu_a & \mu_b & 0 & 0 \\ -2\lambda_a & (S+2\lambda_b+\mu_a+\alpha\lambda_a) & 0 & 0 & -\mu_b \\ -2\lambda_b & 0 & (S+2\lambda_a+\alpha\lambda_b+\mu_b) & -\mu_a & 0 \\ -2\lambda_b & 0 & 0 & (S+\alpha\lambda_a+\alpha\lambda_b+\mu_a) & 0 \\ -2\lambda_a & 0 & 0 & 0 & (S+\alpha\lambda_b+\alpha\lambda_a+\mu_b) \end{bmatrix}$$

MEAN TIME TO SYSTEM FAILURE

The Laplace transform of the reliability of the system is given by:

$$R(s) = P_0(s) + P_1(s) + P_2(s) + P_3(s) + P_4(s)$$

$$R(s) = \frac{4\lambda_a\lambda_b\left(2\lambda_a\mu_a + 2\lambda_b\mu_b - \mu_a\mu_b + B(\mu_a + D)\right) + A\left(2\lambda_a\left(2\lambda_b\mu_b + BD\right) + C\left(4\lambda_a\lambda_b B\left(2\lambda_b + D\right)\right)\right)}{\prod_{r=1}^{5}(s - s_r)} \quad (13)$$

Where, $A = (S + x_1), B = (S + x_2), C = (S + x_3), D = (S + x_4)$

The mean time to failure of the system is given by:

$$MTTF = \lim_{S\to 0} R(S) \begin{array}{l} -4\lambda_A\lambda_B\mu_B^2 + \chi_1\chi_2\chi_3\chi_4 - 2\lambda_A\left(-\chi_1\left(2\lambda_B\left(\alpha\lambda_A + 2\lambda_B + \mu_A\right) - 4\lambda_A\lambda_B\mu_B\right)\right) \\ +\chi_2\chi_1 2\lambda_B\left(\alpha\lambda_A + 2\lambda_B + \mu_A\right) + 4\lambda_A\lambda_B\mu_B + \chi_1\left(4\lambda_A\lambda_B\mu_B - \chi_2\left(-2\lambda_A\left(\lambda_A - 2\beta\lambda_B - \mu_B\right)\right)\right) \\ -2\lambda_B\left(-4\lambda_A\lambda_B\mu_B + \chi_2\left(-2\lambda_A\left(\lambda_A - 2\beta\lambda_B - \mu_B\right)\right)\right) / -4\lambda_A\lambda_B\left(\alpha\lambda_A\lambda_B\mu_A + \beta\lambda_B^2\mu_A + 2\lambda_A\lambda_B + 2\lambda_B^2\mu_B\right) \\ +\chi_2\left(-4\lambda_A\lambda_B\mu_B^2 + \chi_1\chi_4\left(-2\lambda_A\mu_A + 2\chi_3\left(\lambda_A + \lambda_B\right)\right) - 2\chi_3\lambda_B\mu_B\right) \end{array}$$

(14)

SYSTEM AVAI LABILITY

The system availability is the probability that the system operates within the tolerances at a given instant of time and is obtained as follows:

$$\frac{dp_0(t)}{dt} + (2\lambda_A + 2\lambda_B)p_0(t) = \mu_A p_1(t) + \mu_B p_3(t),$$

$$\frac{dp_1(t)}{d(t)} + (\alpha\lambda_A + 2\lambda_B + \mu_A)p_1(t) = 2\lambda_A p_0(t) + \mu_A p_7(t) + \mu_B p_4(t),$$

$$\frac{dp_2(t)}{dt} + (\beta\lambda_B + 2\lambda_A + \mu_B)P_2(t) = 2\lambda_B P_0(t) + \mu_A P_3(t) + \mu_B P_{10}(t),$$

$$\frac{dP_3(t)}{dt} + (\mu_A + \beta\lambda_B + \alpha\lambda_A)P_3(t) = 2\lambda_B P_1(t) + \mu_B P_5(t),$$

$$\frac{dP_4(t)}{dt} + (\beta\lambda_B + \alpha\lambda_A + \mu_B)P_4(t) = 2\lambda_A P_2(t) + \mu_A P_8(t),$$

$$\frac{dP_5(t)}{dt} + (\mu_B)P_5(t) = \beta\lambda_B P_4(t),$$

$$\frac{dP_6(t)}{dt} + \mu_B P_6(t) = \alpha\lambda_A P_4(t),$$

$$\frac{dP_7(t)}{dt}+\mu_A P_7(t)=\alpha\lambda_A P_1(t)+\mu_B P_6(t),$$
$$\frac{dP_8(t)}{dt}+\mu_A P_8(t)=\alpha\lambda_A P_3(t),$$
$$\frac{dP_9(t)}{dt}+\mu_A P_9(t)=\beta\lambda_B P_3(t),$$
$$\frac{dP_{10}(t)}{dt}+\mu_B P_{10}(t)=\beta\lambda_B P_2(t)+\mu_A P_9(t),$$
(15-25)

Using initial conditions, $P_0(0)=1, P_i(0)=0$, where $i=1,2,..,10$. Taking Laplace transforms, of the equations (15-25) and determinant for the following matrix,

$$\left[\begin{array}{ccccccccccc}
s+x1 & -\mu a & -\mu b & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2\lambda a & s+x2 & 0 & 0 & -\mu b & 0 & 0 & -\mu a & 0 & 0 & 0 \\
-2\lambda b & 0 & s+x3 & -\mu a & 0 & 0 & 0 & 0 & 0 & 0 & -\mu b \\
0 & -2\lambda b & 0 & s+x4 & 0 & -\mu b & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -2\lambda a & 0 & s+x5 & 0 & 0 & 0 & -\mu a & 0 & 0 \\
0 & 0 & 0 & 0 & -\beta\lambda b & s+\mu b & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\alpha\lambda a & 0 & s+\mu b & 0 & 0 & 0 & 0 \\
0 & -\alpha\lambda a & 0 & 0 & 0 & 0 & -\mu b & s+\mu a & 0 & 0 & 0 \\
0 & 0 & 0 & -\alpha\lambda a & 0 & 0 & 0 & 0 & s+\mu a & 0 & 0 \\
0 & 0 & 0 & -\beta\lambda b & 0 & 0 & 0 & 0 & 0 & s+\mu a & 0 \\
0 & 0 & -\beta\lambda b & 0 & 0 & 0 & 0 & 0 & 0 & -\mu a & s+\mu b
\end{array}\right.$$

where

$x_1=(2\lambda_A+2\lambda_B)$ $\quad x_2=(\alpha\lambda_A+2\lambda_B+\mu_A)$

$x_5=(\mu_B+\alpha\lambda_A+\beta\lambda_B)$ $\quad x_4=(\mu_A+\alpha\lambda_A+\beta\lambda_B)$ $\quad x_3=(\mu_B+2\lambda_A+\beta\lambda_B)$

Now taking inverse Laplace transforms of Equations $P_0(s), P_1(s), P_2(s), P_3(s)$ and $P_4(s)$ in the matrix

Since S_0, S_1 S_2, S_3 and S_4 correspond to system up-states, the system availability is given by

$$AV(t)=P_0(t)+P_1(t)+P_2(t)+P_3(t)+P_4(t)$$

Behavior of the system from the graphs

Figure (2-4) demonstrate the following results which are only to be expected.

As both the time for taking a unit α and λ_B increases:

1-the mean time to system failure with two sup system A and B increases.

Figure (5) show that the present of additional α lead to improve the values of the mean time to system failure are increases by using two sup system A and B increases as shown from their behaviors when plotted against α.

Figure (6-7) demonstrate the following results which are only to be expected.

As both the time for taking a unit α and λ_B increases:

1-the mean time to system failure with two sup system A and B increases.

Figure (8) show that the present of additional α lead to improve the values of the steady state availability are increases by using two sup system A and B increases as shown from their behaviors when plotted against α.

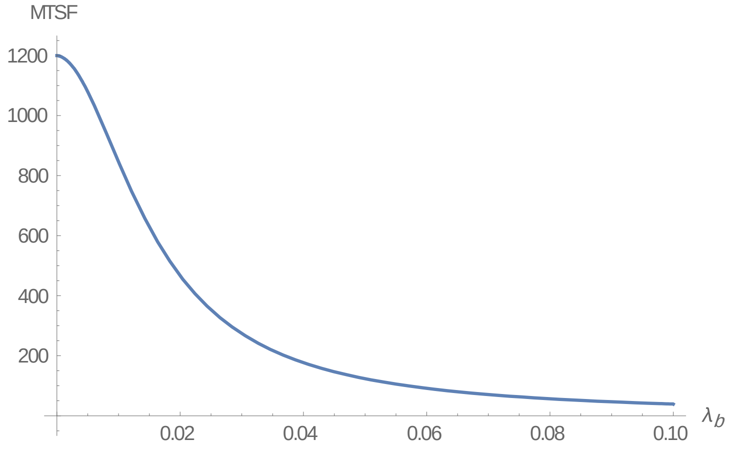


Figure 2. Behavior of the mean time to system failure w.r.t(λ_B***)***

i.e.($\mu_A = \mu_B = 0.9, \lambda_A = 0.02, \beta = \alpha = 1$)

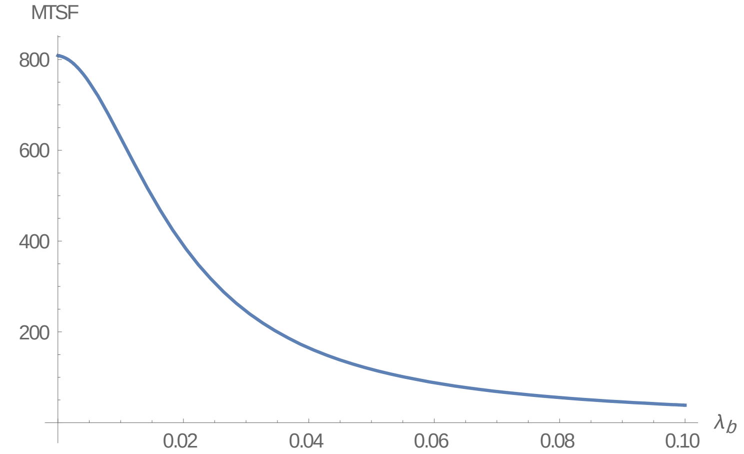


Figure 3. Behavior of the mean time to system failure w.r.t(λ_B***)***

i.e.($\mu_A = \mu_B = 0.9, \lambda_A = 0.02, \beta = 1, \alpha = 1.5$)

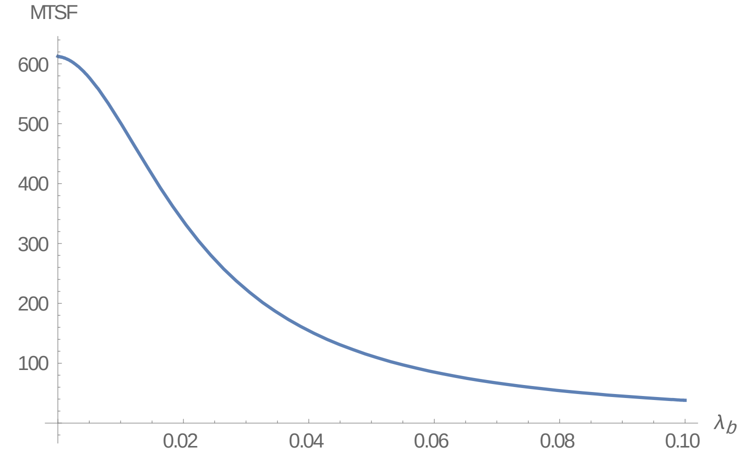


Figure 4. Behavior of the mean time to system failure w.r.t(λ_B***)***

i.e.($\mu_A = \mu_B = 0.9, \lambda_A = 0.02, \beta = 1, \alpha = 2$)

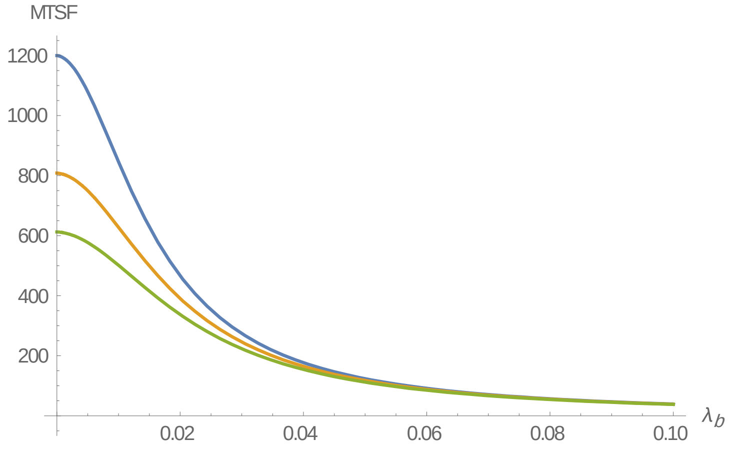


Figure 5. Behavior of the mean time to system failure w.r.t(λ_B***)***

i.e.($\mu_A = \mu_B = 0.9, \lambda_A = 0.02, \beta = 1, \alpha = 1, \alpha = 1.5, \alpha = 2$)

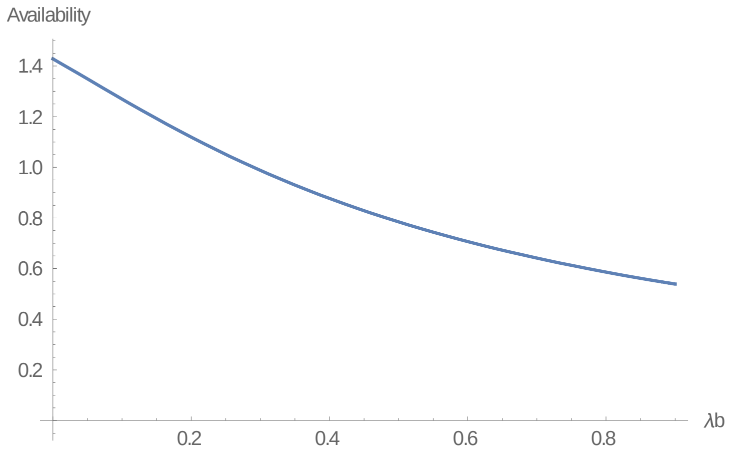


Figure 6. Behavior of the mean time to system failure w.r.t(λ_B***)***

i.e.($\mu_A = \mu_B = 0.9, \lambda_A = 0.02, \beta = 1, \alpha = 1$)

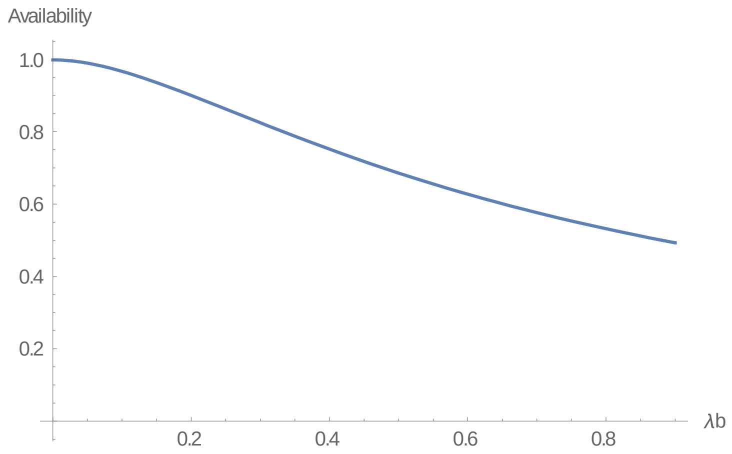


Figure 7. Behavior of the mean time to system failure w.r.t(λ_B***)***

i.e.($\mu_A = \mu_B = 0.9, \lambda_A = 0.02, \beta = 1, \alpha = 1.5$)

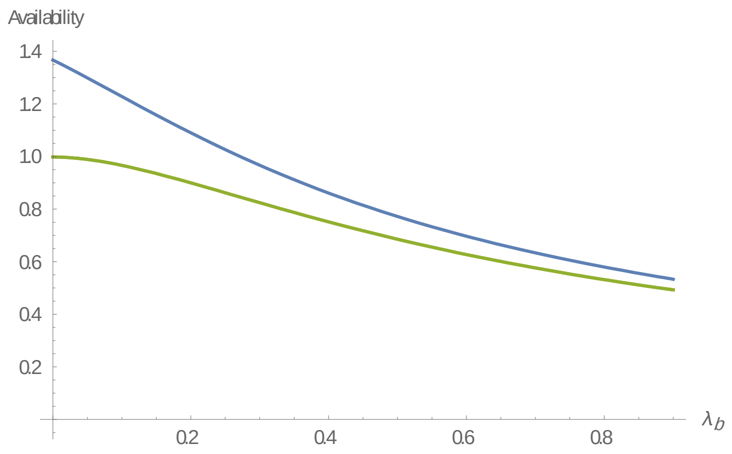


Figure 8. Behavior of the mean time to system failure w.r.t(λ_B)

i.e.($\mu_A = \mu_B = 0.9, \lambda_A = 0.02, \beta = 1, \alpha = 1, \alpha = 1.5$)

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التحليل الاحتمالي لنظام مكون من تركيبة متكررة للوحدات المختلفة الموصلة القابلة للإصلاح

إنتصار السايح

المعهد العالي للعلوم والتقنية، العزيزية، ليبيا

المستخلص

نرغب من هذه الدراسة طريقة للوصول الى اعلى درجات الصلاحية لنظام قابل للإصلاح مكون من وحدتين (A,B) متماثلتين موصلتين على التوالي .علما ان الوحدة A مكونة من وحتين جزيتين موصلتين على التوازي كل منهما تتعرض لنوعين من الخطأ (خطأ بشري ,خطأ الالة)وكذلك الوحدة B تتكون من وحدتين موصلتين على التوازي كل منهما تتعرض لنوعين من الخطأ (خطأ بشري ,خطأ الالة)ومن بعد اعتماد الرسم البياني لهذا النظام نحاول استنتاج المعادلات التفاضلية الانتقالية وثم حل هذه المعادلة باستخدام المصفوفات ثم الحصول على الدالة الاحتمالية واحتمال تعرض النظام لخطأ الالة علما بان النظام يتوقف عن العمل عند فشل الوحدة A كليا او فشل الوحدة B كليا . وباعتبار معدل الفشل والتصليح للوحدتين متغيرات عشوائية يتبعان التوزيع الاسي الثابت ونحاول الوصول الي حساب العلاقة بين معدل الفشل والنفاعية خلال الزمن وتمثيل ذلك بيانيا. كذلك حساب العلاقة بين احتمال فشل النظام الناتج من خطأ الالة ومعدل فشل الوحدة A نتيجة خطأ الالة. كذلك حساب العلاقة بين احتمال فشل النظام الناتج من الخطأ البشري ومعدل فشل الوحدة A نتيجة خطأ بشري. (يتم اشتقاق تحويلات لا بلاس في حل المعادلات الخاصة بالنظام).

الكلمات الدالة. التحليل الاحتمالي، النظام المعقد المكرر، عملية العكس.