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Original article

On Fuzzy Subhypernear-Ring

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ABSTRACT

In this study, two main goals were achieved. In near-ring N, we have proven that, the level set is a subnear-ring of N if and only if the fuzzy set is the fuzzy subnear-ring of N. Similarly, in hyper near-ring R, it has been proven that, a fuzzy set is a fuzzy hypernear-ring if and only if level set is a sub-hypernear-ring of R. A direct proof method was used to reach these results, which will contribute to expanding the field of study on fuzzy near-ring and fuzzy hyper near-rings.

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INTRODUCTION

The fuzzy sets on hyperstructures were first introduced by Zadeh in 1965 [1]. The fuzzy sets and algebraic hyperstructures have been considered by, Abou-Zaid [2], Davvaz [3] and others. Fuzzy hyperideals of hypernear-rings are a notion that Davvaz presented along with certain associated properties.

Fuzzy hyperideals of hypernear-rings are a notion that Davvaz presented along with certain associated properties. In this work, we will discuss relationships between fuzzy subset and level set on near-ring (hypernear-ring) respectively.

Near-ring

Definition 1.1.[6] A ***left near-ring*** is an algebraic structure (N, + , .) which satisfies the following axioms:

(i) (N, +) is a group (not necessarily abelian),
(ii) with respect to the multiplication, (N, .) is a semi-group,
(iii) the multiplication is distributive with respect to the addition on the left side, i.e. , z .(x + y) = z . x + z. y for all x, y, z ∈ N

Or right near-ring if satisfies the right distributive law.

$$(x + y) . z = x .z + y . z, \text{ for all } x, y, z \in N.$$

The term "near-ring" will be used to refer to "left near-ring."

Example 1.2. (Z_8, +) is a group under '+' modulo 8.
Define '.' on Z_8 by a . b = a for all a,b $\in Z_8$. Clearly (Z_8, +, .) is a near-ring.

Definition 1.3.[4] A subgroup M of an near-ring N with $M.M \subseteq M$ is called a ***subnear-ring*** of N, (M ≤ N). A subgroup S of N with $N.S \subseteq S$ is called a ***normal subgroup*** of N,($S \trianglelefteq N$).

Hyper near-ring

Definition 2.1.[6] Let H be a nonempty set. A map ∘ : H × H → P*(H) is called ***hyper-operation***, P*(H) is the family of all nonvoid subsets of H.

Definition 2.2.[9] The triple (R , + , .) is a ***hypernear-ring*** if:

I) (R,+) satisfies the following axioms:
(1) x + (y + z) = (x + y) + z, for any x, y, z ∈ R.

(2) ∃ 0 ∈ R s.t. for any x ∈ R, x + 0 = 0 + x = x.
(3) for any x ∈ R, there exists a unique element –x ∈ R, such that
$0 \in x+(-x) \cap -x + x$.
(4) for any x,y,z ∈R, z ∈ x+y implies that x ∈ z+ (−y) , y ∈ − x +z.

II) (R , ·) is a semi-group endowed with a two-sided absorbing element 0, i.e . for any x ∈ R , x·0 = 0·x =0.

III) The operation ' · ' is distributive with respect to the hyperoperation ' + ' from the left side: For many x,y,z ∈R, x· (y+z) = x·y + x·z,

Definition 2.3.[7] Let (R,+, ·) be a hypernear-ring. A non-empty subset H of R is called a subhypernear-ring if
(1) (H, +) is a subhypergroup of (R, +), i.e., a, b ∈ H implies a + b ⊆ H, and a ∈ H implies −a ∈ H,
(2) ab∈ H, for all a, b ∈ H.

Example 2.4.[9] Let R = {0, a, b, c} be a set with a hyperoperation "+" and a binary operation "·" as follows:

Table 2.1

+	0	a	b	c
0	{0}	{a}	{b}	{c}
a	{a}	{0,a}	{b}	{c}
b	{b}	{b}	{0,a,c}	{b,c}
c	{c}	{c}	{b,c}	{0,a,b}

Table 2.2

.	0	a	b	c
0	0	a	b	c
a	0	a	b	c
b	0	a	b	c
c	0	a	b	c

Then (R,+, ·) is a hypernear-ring, and {0}, {0, a}, and R are subhypernear-rings of R.

Fuzzy structure

Definition 3.1.[8] A ***fuzzy subset*** of X is a function μ: X → [0, 1]. The set of all fuzzy subsets of X is called the fuzzy power set of X and is denoted by $\mathcal{FP}$(X). For t ∈ [0, 1], define μ_t as follows:

$$\mu_t = \{x \mid x \in X, \mu(x) \geq t\}.$$ μt is called the ***t-level set*** of μ.

Example 3.2. In Example 2.4. R = {0, a, b, c}, define a fuzzy subset μ:R→[0,1] by : μ (0)=1, μ(a) = 0.7, μ(b)= μ(c) = 0.3 .
Note that $\mu_{0.45}$= {x ∈U | μ(x) ≥ 0.45} = {0, a} and μ_0= R.

Definition 3.3.[7] Let N be a near-ring and μ be a fuzzy subset of N. We say μ a ***fuzzy subnear-ring of N*** if for all x, y∈N,
(1) μ(x − y) ≥ min {μ(x) , μ(y)},
(2) μ(xy) ≥ min {μ(x) , μ(y)} .

Theorem 3.4. Let N be a near-ring and μ be a fuzzy subset of N. Then the level subset μ_t ($\neq \phi$) is a subnear-ring of N for all t ∈ (0, 1] if and only if μ is a fuzzy subnear-ring of N.

Proof. Let μ_t is a subnear-ring of N .
Let x , y ∈N and putting t_0 = min {μ (x), μ (y)} then x, y $\in \mu_{t_0}$
⇒ μ (x) ≥ t0 , μ (y) ≥ t_0 , and since μt is a subnear-ring of N , hence xy$\in \mu_{t_0}$,and so μ (xy)≥ t0 = min {μ (x), μ (y)}, also , x – y $\in \mu_{t_0}$ so
μ (x – y)≥ t_0 = min {μ (x), μ (y)}, therefore μ is fuzzy subnear-ring.

Conversly , Let μ is fuzzy subnear-ring and t ∈ (0, 1] , μt ($\neq \phi$)
Let x, y ∈μ_t ⇒ μ (x) ≥ t , μ (y) ≥ t , then min {μ (x) , μ (y)} ≥ t .
And μ (xy) ≥ min {μ (x), μ (y)} ≥ t (since μ is fuzzy subnear-ring)
Thererfore xy ≥μ_t , hence μ_t. $\mu_t \subseteq \mu_t$. Thus, μ_t is a subnear-ring of N. ■

Definition 3.5.[5] Let (R, +, ·) be a hypernear-ring. Then we call a fuzzy set μ of R a ***fuzzy subhypernear-ring of R*** if it satisfies the following inequalities:
(1a) min {μ(x) , μ(y) } ≤ $\inf_{z\in x+y}\mu(z)$ for all x, y ∈ R,
(1b) μ(x) ≤ μ(- x) for all x ∈ R,
(2) min {μ(x) , μ(y) } ≤ μ(xy) for all x, y ∈ R.

Theorem 3.6. A fuzzy set μ of R is a fuzzy subhypernear-ring of R if and only if for any t ∈ [0, 1] , μ_t ($\neq \phi$) is a sub-hypernear-ring of R.

Proof. Let μ is a fuzzy sub-hypernear-ring and t ∈ [0 , 1] , $\mu_t \neq \emptyset$

If x , y $\in \mu_t \Longrightarrow \mu(x) \geq t$, $\Longrightarrow \mu(y) \geq t$
Hence inf $\mu(z) \geq \min\{\mu(x), \mu(y)\} \geq t$ where z $\in$ x + y
Therefore, for all z $\in$ x + y we have z $\in \mu_t$ and so x + y $\subseteq \mu_t$
Thus μ_t is sub-hypernear-ring of R

Conversely, let μ_t is sub-hypernear-ring of R
Let x , y $\in$ R and putting t0= min{μ (x) , μ (y)} then x , y $\in \mu_{t_0}$
Since μ_t is sub-hypernear-ring of R
$\therefore$ x + y $\subseteq \mu_t \Longrightarrow$ for any z $\in$ x + y, z $\in \mu_{t_0}$ which implies that
inf $\mu(z) \geq t_0 = \min\{\mu(x), \mu(y)\}$ where z $\in$ x + y
and since μt is a sub-hypernear-ring of R, hence xy $\in \mu_t$

$$\mu(xy) \geq t_0 = \min\{\mu(x), \mu(y)\}$$

Thus μ of R is a fuzzy subhypernear-ring of R.■

Example 3.7. In Example 2.4., and Example 3.2.

$$\mu t = \begin{cases} R & , t \in (0,0.3] \\ \{0, a\} & , t \in (0.3,0.7] \\ \{0\} & , t \in (0.7,1] \end{cases}$$

Clearly, all μt are subhypernear-ring of R, and μ is a fuzzy subhypernear-ring of R.

CONCLUSION

In this paper, the following has been proven that the level set of near-ring is a subnear-ring if and only if the fuzzy set is the fuzzy subnear-ring. A fuzzy set of hyper near-ring is a fuzzy hypernear-ring if and only if level set is a sub-hypernear-ring.

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حول قريب الحلقة الفوقية الجزئية الضبابية

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المستخلص:
في هذه الدراسة، تم تحقيق هدفين، في قريب الحلقة، تم أثبات أن، المجموعة المنسوبية تكون قريب الحلقة الجزئية لقريب الحلقة اذاواذا فقط كانت المجموعة الضبابية هي قريب الحلقة الضبابية. بالمثل في قريب الحلقة الضبابية تم أثبات أن، المجموعة الضبابية تكون قريب الحلقة الضبابية اذا واذا فقط كانت المجموعة المنسوبية هي قريب الحلقة الفوقية الجزئية لقريب الحلقة الفوقية. تم استخدام أسلوب البرهان المباشر للوصول لهذه النتائج، التي ستسهم في توسيع مجال الدراسة حول قريب الحلقات الضبابية والفوقية الضبابية.
الكلمات الدالة: قريب الحلقة، قريب الحلقة الجزئية، المجموعة المنسوبية، المجموعة الضبابية، قريب الحلقة الفوقية، قريب الحلقة الفائقة الضبابية.