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Original article

Laplace Adomian Decomposition Method for Fractional Order SIS Epidemic Model

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ABSTRACT

This research manuscript addresses finding a solution to the SIS Epidemic model using the Caputo derivative. To achieve the required results, Laplace transform together with the Adomian Decomposition Method (LADM) were utilized. This method is a powerful tool for dealing with various linear and nonlinear problems of fractional-order differential equations (FODEs). Additionally, some results related to the qualitative theory of the model of interest were studied. The computational results showed the verification of the performed analysis. The investigation of the approximate solution computed in the form of an infinite series was carried out. The results were graphically displayed to analyze the adopted procedure for solving nonlinear FODEs using the Caputo fractional derivative

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INTRODUCTION

The use of mathematical expressions provides a fundamental description of physical reality, enabling researchers to enhance their understanding of underlying phenomena by studying accurate or approximate solutions of mathematical models [1]. However, finding analytical solutions to these complex mathematical models often poses significant challenges. As such, the development of robust numerical solutions has become highly advantageous for solving these intricate problems. Fractional derivatives have emerged as a powerful tool, offering researchers new avenues for modeling a wide range of phenomena across various scientific disciplines [2]. The primary motivation for utilizing fractional derivatives, rather than their integer-order counterparts, is the ability to account for the effects of memory in the modeling process. Integer-order models often fail to capture the memory effects that can have a profound impact on the dynamics of complex systems, such as human populations in the context of disease transmission.

By incorporating fractional derivatives, researchers can develop more comprehensive and accurate mathematical models that better reflect the complex behaviors observed in real-world systems. This approach has the potential to lead to a deeper understanding of the underlying mechanisms governing these phenomena and facilitate the development of more effective strategies for addressing the challenges they present.

The study of fractional-order models and their analytical or numerical solutions has emerged as a vibrant and rapidly evolving field of research, with widespread applications across the scientific landscape. Continued advancements in this area hold the promise of transformative insights and innovative solutions to the complex problems faced by the research community.

In the context of disease transmission dynamics, fractional derivatives offer a compelling approach to account for the memory effect within populations. The memory effect refers to the influence of past events on the current state of the system. In the case of disease spread, this memory effect can represent factors such as acquired immunity, previous exposure, or behavioral changes influenced by past experiences. By considering these memory-related effects through

the use of fractional derivatives, researchers can develop more accurate and comprehensive models that better capture the underlying dynamics of disease propagation [3,4].

The incorporation of fractional derivatives provides researchers with a powerful mathematical tool for modeling and understanding complex systems in which memory and non-local behavior play a significant role - aspects that cannot be adequately captured by traditional integer-order models. These derivatives offer a means to bridge the mathematical description and the inherent complexities of real-world phenomena, leading to improved predictions, analyses, and the development of more effective control strategies across various scientific disciplines [5,6].

The ability of fractional derivatives to account for memory effects is particularly beneficial in the context of disease transmission modeling. By considering the influence of past events, such as previous exposures and acquired immunity, researchers can develop more realistic and accurate models that better reflect the true dynamics of disease spread within human populations. This, in turn, can lead to enhanced understanding of disease transmission patterns, the development of more targeted intervention strategies, and the formulation of more effective public health policies.

Furthermore, the application of fractional derivatives extends beyond the realm of disease transmission, providing a versatile mathematical framework for modeling and analyzing a wide range of complex systems in fields like physics, engineering, biology, and economics, where memory and non-local behavior are prevalent. The continued advancements in the theory and applications of fractional calculus hold great promise for addressing the challenges faced by researchers in these diverse domains.

The wide-ranging applicability of fractional calculus has been extensively documented in the literature [7]. In a seminal work, Ross presented the crucial criteria that define fractional derivatives, establishing a robust mathematical framework for this field [8]. Researchers have been actively exploring various analytical and numerical techniques to solve both linear and nonlinear fractional differential equations. Some have adapted classical methods to enhance their effectiveness, while others have established connections between two or more techniques to obtain solutions for fractional equations. One of the prominent models in the study of infectious disease dynamics is the Susceptible-Infected-Susceptible (SIS) epidemic model. This model describes the dynamic interaction between individuals who are susceptible to infection and those who are currently infected, particularly applicable to diseases that do not confer permanent immunity after infection, such as the common cold and influenza.

The SIS model represents the transition of individuals between the susceptible and infected states, which can be characterized by two key rates: the rate of transition from the susceptible state to the infected state (through the process of infection) and the rate of transition from the infected state to the susceptible state (through recovery or the loss of acquired immunity) [9].

The use of fractional derivatives in the context of the Susceptible-Infected-Susceptible (SIS) epidemic model has gained significant attention, as it allows for the incorporation of memory effects and non-local behavior often observed in the spread of infectious diseases. By employing fractional-order derivatives, researchers can develop more comprehensive and accurate models that capture the complex dynamics underlying disease transmission within human populations. The overarching goal of this work is to contribute to the expanding body of mathematical tools and techniques available for the rigorous modeling and analysis of infectious disease dynamics, as exemplified by the SIS epidemiological framework. The findings of this study have the potential to enhance our fundamental understanding of disease transmission processes and inform the development of more effective intervention strategies.

Related Work

The SIS model describes the dynamic interaction between susceptible and infected individuals, representing the transition between these two states. Key factors affecting the epidemic dynamics in this model are the infection rate and the recovery or immunity loss rate [10]. The model can be used to estimate the spread of the infection, the duration of the epidemic, and the impact of changes in the infection rate or recovery rate on the numbers of susceptible and infected individuals.

Considered one of the fundamental models in epidemiology and medical research, the SIS model helps to understand the basic dynamics of infectious disease spread and analyze the effects of preventive interventions, such as vaccination or public health measures [11].

$$\frac{dS(t)}{dt} = \mu N - \beta S(t) + \gamma I(t) - \mu S(t).$$

$$\frac{dI(t)}{dt} = \beta S(t) I(t) - (\mu + \gamma) I(t).$$

Adomian Decomposition Method (ADM) is a well-established numerical technique that has been extensively employed to find approximate solutions to various mathematical models, including those describing epidemic dynamics [12]. Proposed by George Adomian, the ADM is a universal method that considers the approximate solution of a nonlinear equation as an infinite series, which typically converges to the exact solution.

One of the primary advantages of the ADM is its ability to solve a wide range of integral and differential equations, including those encountered in the context of epidemic modeling. This versatility makes the ADM a valuable tool for researchers investigating the dynamics of infectious disease spread using mathematical models, such as the Susceptible-Infected-Susceptible (SIS) model.

In comparison to other numerical methods, such as the Homotopy Analysis Method (HAM) and the Differential Transformed Method (DTM), the ADM offers a unique and systematic approach to solving nonlinear problems. By decomposing the nonlinear terms into a series of Adomian polynomials, the ADM provides a straightforward and efficient way to obtain approximate solutions, even for complex mathematical models.

Building upon the strengths of the ADM, the Laplace Adomian Decomposition Method (LADM) has been proposed as an advanced mathematical approach for analyzing differential equations and their applications in various scientific and engineering fields [13,14]. The LADM combines the Laplace transform and the Adomian decomposition to solve fractional-order differential equations with infinite kernels.

The LADM relies on the Laplace transform to convert the ordinary or partial differential equation into an algebraic equation in the frequency domain, and then applies the Adomian decomposition analysis to solve the algebraic equation and retrieve the time-domain solution of the original differential equation. The Adomian analysis method is employed to handle the infinite kernel and challenges arising from the fractional order of the differential equation.

The Laplace Adomian Decomposition Method (LADM) holds significant importance in the fields of numerical computation and mathematical analysis, as it provides a semi-analytical solution approach for solving complex differential equations [15,16]. This advanced technique aids in the understanding of system characteristics and dynamic behavior, making it a valuable tool for researchers and engineers working in various domains.

The Laplace Adomian Decomposition Method (LADM) has emerged as a powerful mathematical approach for solving complex differential equations, enabling researchers and engineers to gain deeper insights into a wide range of physical, chemical, and biological systems. By combining the Laplace transform and the Adomian decomposition analysis, the LADM offers a versatile means of tackling fractional-order differential equations, which are commonly encountered in modeling diverse phenomena.

The effectiveness of the LADM has been demonstrated across various disciplines, including biological sciences, electrical engineering, and materials science. This advanced technique provides a semi-analytical solution approach, allowing for the elucidation of complex system characteristics and dynamic behavior. The LADM's ability to derive high-precision solutions has made it a valuable tool for researchers and engineers seeking to understand and predict the behavior of intricate systems.

At the core of the LADM's efficacy lies its unique integration of the Laplace transform and the Adomian decomposition analysis. The Laplace transform is employed to convert the original ordinary or partial differential equation into an algebraic equation in the frequency domain, while the Adomian decomposition is used to solve this algebraic equation and retrieve the corresponding time-domain solution. This combined approach enables the LADM to effectively handle the infinite kernel and challenges associated with the fractional order of the differential equation.

Despite the significant advantages of the LADM, its effective implementation requires a strong foundation in mathematical analysis and numerical computation [16,17]. It is essential to develop efficient and suitable software tools capable of accurately performing Laplace transformations and Adomian decomposition analyses, in order to ensure the reliable and accurate application of this method.

The Laplace Adomian Decomposition Method (LADM) represents a significant advancement in the field of fractional-order differential equation solving, characterized by its comprehensive applicability and demonstrated effectiveness across multiple scientific and engineering domains. The continued development and refinement of this method, along with the creation of robust software tools, will further enhance its utility and impact in the.

METHOD

Fractional-Order Epidemiological Model with Laplace Adomian Decomposition Analysis

This section is dedicated to the analysis of the fractional-order epidemiological model (1) with specified initial conditions. To investigate this model, we apply the Laplace transform to both sides of the governing equations, which allows for further mathematical analysis and solution derivation.

The fractional-order epidemiological model under consideration is given by:

$$
\begin{aligned}
{}^{c}D_t^{\alpha} S(t) &= \mu N - \beta S(t) + \gamma I(t) - \mu S(t).\\
{}^{c}D_t^{\alpha} I(t) &= \beta S(t) I(t) - (\mu + \gamma) I(t). \qquad (1)
\end{aligned}
$$

with the initial conditions:

$$S(0) = n, I(0) = m, and\ S(t) + I(t) = N(t) = 1$$

${}^{c}D_t^{\alpha}$ represents the Caputo fractional derivative of order $\alpha \in (0,1]$, μ denotes the birth and death rate, β is the contact rate, γ is the recovery rate, and $S(t)$ and $I(t)$ are the time-dependent proportions of susceptible and infected individuals, respectively, within a continuous population $N(t) = 1$, [21].

To the best of our knowledge, an explicit analytical solution formula for this fractional-order epidemiological model is not readily available in the existing. Therefore, we employ a series representation approach based on the Laplace Adomian Decomposition Method (LADM) to derive explicit expressions for the unknown functions S(t) and I(t).

By applying the Laplace transform to the fractional-order differential equations, we obtain a transformed system of algebraic equations that can be solved using the LADM. This approach allows us to construct a convergent series solution for the susceptible and infected populations, which can be evaluated numerically to obtain accurate approximations.

Furthermore, we validate the accuracy of the theoretical LADM-based solution formulas by comparing them with the results of two distinct numerical schemes: the Grunwald-Letnikov method and the Adomian Decomposition Method (ADM) [17,18]. Additionally, we investigate the impact of the fractional derivative order α on the system dynamics by analyzing the behavior of the solutions as α approaches the integer value of 1, which corresponds to the standard epidemiological model.

The derivation of the explicit LADM-based solution formulas, along with the numerical validations and the comparative analysis of the fractional and integer-order models, provide valuable insights into the dynamics and applications of this fractional-order epidemiological system [17,20].

Applying the Laplace transform on both sides of (1),we get

$$
\begin{aligned}
\mathcal{L}\{\, {}^{c}D_t^{\alpha_1} S(t)\} &= \mathcal{L}\{\mu N - \beta S(t) + \gamma I(t) - \mu S(t)\}\\
\mathcal{L}\{\, {}^{c}D_t^{\alpha_2} I(t)\} &= \mathcal{L}\{\beta S(t) I(t) - (\mu + \gamma) I(t)\} \qquad (2)
\end{aligned}
$$

using the property of Laplace transform, we have

$$
\begin{aligned}
s^{\alpha_1}\mathcal{L}\{S(t)\} - s^{\alpha_1 - 1} S(0) &= \mathcal{L}\{\mu N - \beta S(t) + \gamma I(t) - \mu S(t)\}\\
s^{\alpha_2}\mathcal{L}\{S(t)\} - s^{\alpha_2 - 1} I(0) &= \mathcal{L}\{\beta S(t) I(t) - (\mu + \gamma) I(t)\} \qquad (3)
\end{aligned}
$$

Now using initial conditions and taking inverse Laplace transform to system (3), we have

$$
\begin{aligned}
S(t) &= \frac{S_0}{s} + \mathcal{L}^{-1}\left\{\frac{1}{S^{\alpha_1}}\, \mathcal{L}\{\mu N - \beta S(t) + \gamma I(t) - \mu S(t)\}\right\}.\\
I(t) &= \frac{I_0}{s} + \mathcal{L}^{-1}\left\{\frac{1}{S^{\alpha_2}}\, \mathcal{L}\{\beta S(t) I(t) - (\mu + \gamma) I(t)\}\right\}. \qquad (4)
\end{aligned}
$$

It should be assumed that method gives the solution as an infinite series

$$S(t) = \sum_{k=0}^{\infty} S_k, \quad I(t) = \sum_{k=0}^{\infty} I_k.$$

and the nonlinear terms involved in the model are $S(t)I(t)$ are decompose by Adomian polynomial as

$$SI = \sum_{j=0}^{\infty} A_k$$

Where A_k are Adomian polynomials defined as

$$A_k = \frac{1}{\Gamma(k+1)} \frac{d^k}{dh^k} \left[\sum_{j=0}^{k} h^k S_j \sum_{j=0}^{k} h^k I_j \right] | h = 0$$

We can calculate the first four terms

$A_0 = S_0 I_0, A_1 = S_0 I_1 + S_1 I_0, A_2 = S_0 I_2 + S_1 I_1 + S_2 I_0, A_3 = S_0 I_3 + S_1 I_2 + S_2 I_1 + S_3 I_0$ ... ...

Substituting the above infinite series form into equation (4), we have

$$\sum_{k=0}^{\infty} S_k(t) = \frac{S_0}{s} + \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_1}}\mathcal{L}\{\mu N - \beta\sum_{k=0}^{\infty} S_k(t) + \gamma\sum_{k=0}^{\infty} I_k(t) - \mu\sum_{k=0}^{\infty} S_k(t)\}\right\}.$$

$$\sum_{k=0}^{\infty} I_k(t) = \frac{I_0}{s} + \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_2}}\mathcal{L}\{\beta\sum_{k=0}^{\infty} A_k(t) - (\mu+\gamma)\sum_{k=0}^{\infty} I_k(t)\}\right\} \quad (5)$$

By matching the terms on both sides of the equation (3.3), we can get the following iterative algorithm

$S_0 = n$, $I_0 = m$.

when $k = 0$, from first equation of (5)

$$S_1 = \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_1}}\mathcal{L}\{\mu N - \beta S_0(t) + \gamma I_0(t) - \mu S_0(t)\}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_1}}\mathcal{L}\{\mu N - \beta n + \gamma m - \mu n\}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_1}}\frac{1}{s}(\mu N - (\beta+\mu)n + \gamma m)\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_1+1}}(\mu N - (\beta+\mu)n + \gamma m)\right\}$$

$$= (\mu N - (\beta+\mu)n + \gamma m)\frac{t^{\alpha_1}}{\Gamma(\alpha_1+1)}$$

when $k = 0$, from second equation of (5)

$I_1 = \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_2}}\mathcal{L}\{\beta A_0(t) - (\mu+\gamma)I_0(t)\}\right\}$.

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_2}}\mathcal{L}\{\beta nm - (\mu+\gamma)m\}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_2}}\frac{1}{s}(\beta nm - (\mu+\gamma)m)\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_2+1}}(\beta nm - (\mu+\gamma)m)\right\}$$

$$= (\beta nm - (\mu+\gamma)m)\frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)}$$

When $k = 1$, from first equation of (5)

$$S_2 = \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_1}}\mathcal{L}\{\mu N - \beta S_1(t) + \gamma I_1(t) - \mu S_1(t)\}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_1}}\mathcal{L}\left\{\mu N - (\beta+\mu)(\mu N - (\beta+\mu)n + \gamma m)\frac{t^{\alpha_1}}{\Gamma(\alpha_1+1)} + \gamma(\beta nm - (\mu+\gamma)m)\frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)}\right\}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_1}}(\mu N - (\beta+\mu)(\mu N - (\beta+\mu)n + \gamma m)\frac{\Gamma(\alpha_1+1)}{\Gamma(\alpha_1+1)s^{\alpha_1+1}} + \gamma(\beta nm - (\mu+\gamma)m)\frac{\Gamma(\alpha_2+1)}{\Gamma(\alpha_2+1)s^{\alpha_2+1}})\right\}$$

$$= \mathcal{L}^{-1}\left\{(\mu N - (\beta+\mu)(\mu N - (\beta+\mu)n + \gamma m)\frac{1}{s^{2\alpha_1+1}} + \gamma(\beta nm - (\mu+\gamma)m)\frac{1}{s^{\alpha_1+\alpha_2+1}})\right\}$$

$$= (\mu N - (\beta+\mu)(\mu N - (\beta+\mu)n + \gamma m)\frac{t^{2\alpha_1}}{\Gamma(2\alpha_1+1)} + \gamma(\beta nm - (\mu+\gamma)m)\frac{t^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1+\alpha_2+1)})$$

When $k = 1$, from second equation of (5)

$$I_2 = \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_2}}\mathcal{L}\{\beta A_1(t) - (\mu+\gamma)I_1(t)\}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_2}}\mathcal{L}\{\beta\left[n(\beta nm - (\mu+\gamma)m)\frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)} + m(\mu N - (\beta+\mu)n + \gamma m)\frac{t^{\alpha_1}}{\Gamma(\alpha_1+1)}\right] - (\mu+\gamma)(\beta nm - (\mu+\gamma)m)\frac{t^{\alpha_2}}{\Gamma(\alpha_2+1)}\}\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^{\alpha_2}}\{\beta\left[(\beta n^2 m - (\mu+\gamma)nm)\frac{\Gamma(\alpha_2+1)}{\Gamma(\alpha_2+1)s^{\alpha_2}} + (\mu Nm - (\beta+\mu)nm + \gamma m^2)\frac{\Gamma(\alpha_1+1)}{\Gamma(\alpha_1+1)s^{\alpha_1}}\right]\right.$$

$$\left. - (\mu+\gamma)(\beta nm - (\mu+\gamma)m)\frac{\Gamma(\alpha_2+1)}{\Gamma(\alpha_2+1)s^{\alpha_2}}\}\right\}$$

$$= \left\{ \beta \left[(\beta n^2 m - (\mu+\gamma)nm) \frac{t^{2\alpha_2}}{\Gamma(2\alpha_2+1)} + (\mu N m - (\beta+\mu)nm + \gamma m^2) \frac{t^{\alpha_1+\alpha_2}}{\Gamma(\alpha_1+\alpha_2+1)} \right] - (\mu+\gamma)(\beta nm - (\mu+\gamma)m) \frac{t^{2\alpha_2}}{\Gamma(2\alpha_2+1)} \right\}$$

$$\vdots$$

Similarly, we can get the rest of the terms by using the system of equation

$$S_{k+1} = \mathcal{L}^{-1}\left\{ \frac{1}{s^{\alpha_1}} \mathcal{L}\{\mu N - \beta S_k(t) + \gamma I_k(t) - \mu S_k(t)\} \right\}$$
$$I_{k+1} = \mathcal{L}^{-1}\left\{ \frac{1}{s^{\alpha_2}} \mathcal{L}\{\beta A_k(t) - (\mu+\gamma) I_k(t)\} \right\} \quad (6)$$

So, we get the solution of the model as an infinite series

$$S(t) = S_0 + S_1 + S_2 + S_3 + \cdots + S_{n+\ldots}$$
$$I(t) = I_0 + I_1 + I_2 + I_3$$

NUMERICAL RESULTS

In this manuscript, some numerical simulations with the Caputo derivative operator for fractional order SIS model (1) are presented using Laplace Adomian Decomposition Method (LADM). Subject to the initial conditions: $S_0 = 620$, $I_0 = 480$, and parameters value $\beta = 0.05,, \mu = 0.02, \delta = 0.3$, For the SIS model, a Caputo fractional order derivative was created and the Laplace Transformation and Adomian Decomposition Method were used to successfully analyze it.

To highlight the efficiency of the purposive approach, the fractional model (1) was solved for $\alpha = 1$, and the numerical solution plots are presented in Figure 1. These plots show good agreement with the results reported in reference [20].

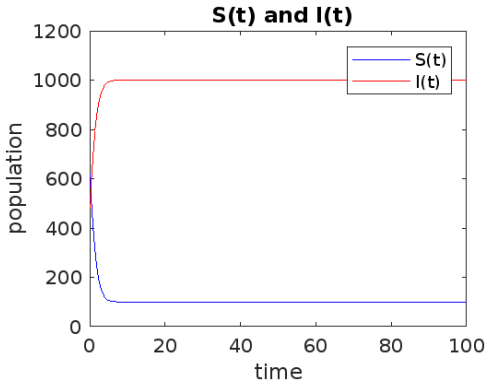

Figure 1. Simulation results for SIS at $\alpha = 1$.

In figure 1 When the epidemic starts, most individuals are susceptible. Once the virus begins to spread, the number of infected individuals increases rapidly because they transmit the infection to susceptible individuals. During the spread, the number of infected individuals increases rapidly, leading to a decrease in the number of susceptible individuals and for equilibrium Point, the system reaches a stable state where the number of infected and susceptible individuals remains constant. At this stage, the rate of infection and recovery is balanced, keeping the number of infected individual's constant.

To study the impact of the fractional order on the approximated state functions of model (1), we analyzed several values of α, and numerical results were documented in Figures 2 and 3. All plots in Figure 2 exhibit a decreasing trend and stabilize after several days.

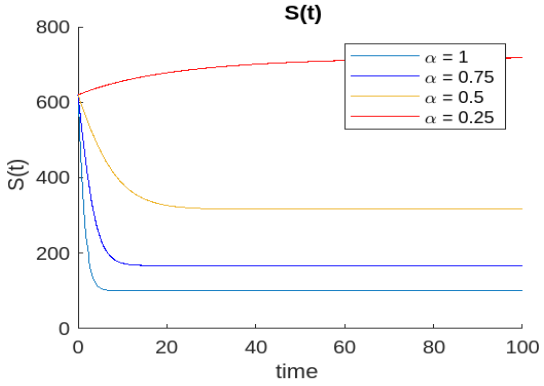

Figure 2. Simulation results for susceptible at different order.

As we can see, for the highest fractional order $\alpha = 1$, Drops rapidly at first, then stabilizes at a low value. This indicates that the system starts with a large number of susceptible individuals, but due to the rapid infection rate, the number quickly decreases to a low stable value. It is observed that when $\alpha = 0.75$,0.5 the same results can be obtained, but at slower rate than $\alpha = 1$, and for $\alpha = 0.25$ Shows the least steep decline and stabilizes at a much higher value. This indicates a very slow infection rate, allowing most individuals to remain susceptible. We can summarize the results as follows:

The higher α, the faster the infection spreads, causing the number of susceptible individuals to drop rapidly and stabilize at a low value. The lower the α , the slower the infection spreads, causing the number of susceptible individuals to decrease slowly and stabilize at a higher value.

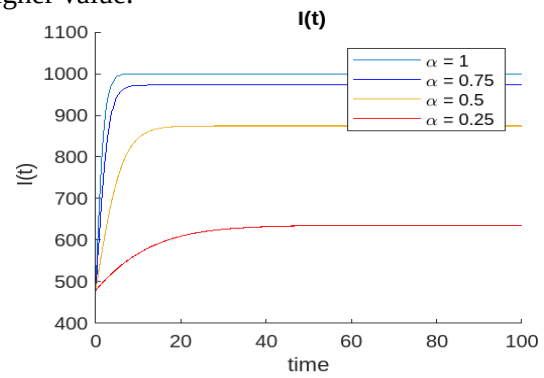

Figure 3. Simulation results for infected at different order.

The graph in fig.3 shows the number of infected individuals $I(t)$ over time for the SIS model with different infection rates α. We can analysis of the Curves for $\alpha = 1$, rises quickly at first and stabilizes at a high value. This indicates that the system starts with a certain number of infected individuals, and due to the rapid infection rate, the number of infected increases quickly and stabilizes at a high value. for$\alpha = 0.75,0.5$, rises less sharply than $\alpha = 1$, and stabilizes at a slightly lower value. This suggests that the infection spreads at a slower rate than $\alpha = 1$, leading to a lower stable number of infected individuals. And for $\alpha = 0.25$, shows the slowest increase and stabilizes at a much lower value. This indicates a very slow infection rate, resulting in a lower stable number of infected individuals. we can summarize as: The graph reflects how the infection rate α affects the disease dynamics in a community. With a higher infection rate, the number of infected individuals increases quickly and stabilizes at a high value, while with a lower infection rate, the number of infected individuals increases slowly and stabilizes at a lower value.

CONCLUSION

In conclusion, this manuscript presents an effective approach to solving the SIS model using the Caputo derivative by employing the Laplace transform in conjunction with the Adomian decomposition method (LADM). The study demonstrates the robustness of LADM in addressing both linear and nonlinear fractional order differential equations (FODEs). Through computational and qualitative analyses, the existence of a solution is affirmed, and an approximate solution is derived in the form of an infinite series. Graphical representations further validate the efficiency of the proposed method in handling nonlinear FODEs under the Caputo fractional derivative, showcasing the method's potential for broader applications in complex differential equation models.

Conflict of interest. Nil

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طريقة تحلل لابلاس أدوميان لنموذج SIS الوبائي ذو الرتبة الكسرية

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المستخلص

تتناول هذه الورقة البحثية إيجاد حل لنموذج SIS الوبائي باستخدام مشتقة كابوتو. لتحقيق النتائج المطلوبة، تم استخدام تحويل لابلاس مع طريقة تحلل أدوميان. تُعتبر هذه الطريقة أداة قوية للتعامل مع المشكلات الخطية وغير الخطية المختلفة للمعادلات التفاضلية الكسرية الرتبة (FODEs). بالإضافة إلى ذلك، تم دراسة بعض النتائج المتعلقة بالنظرية النوعية للنموذج محل الاهتمام. تم التحقيق في الحل التقريبي المحسوب في شكل سلسلة لانهائية. تم عرض النتائج بشكل بياني لتحليل الإجراءات المعتمدة لحل المعادلات التفاضلية الكسرية غير الخطية باستخدام مشتقة كابوتو الكسرية.

الكلمات المفتاحية. معادلة تفاضلية ذات رتبة كسرية، حل تحليلي، طريقة تحلل لابلاس أدوميان، نموذج SIS.