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Original article

Duality Between the Abaoub-Shkheam Transform and Some Important Integral Transforms

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Abstract
Nowadays, integral transforms are the most appropriate techniques for finding the solution of typical problems because these techniques convert them into simpler problems. Finding the solution of initial value problems is the main use of integral transforms. Also, there are so many other applications of integral transforms in different areas of mathematics, statistics, and engineering, such as in solving improper integrals of the first kind, finding the solution of heat transfer problems, electrical networks, and signal processing problems. In this research paper, we will discuss connections between the Abaoub-Shkheam transform, a new integral transform type, and some effective integral transforms. Integral transforms of some typical functions are presented in a table to effectively illustrate the relationship between the Abaoub-Shkheam transform and some effectively mentioned integral transforms.
Keywords. Integral Transforms, Dualities, Abaoub-shkheam Transform, Laplace Transform, El-zaki Transform.

Introduction

Integral transformations are mathematical tools for solving ordinary and partial differential equations and special types of integral equations [1,2]. Therefore, this field has recently gained great attention from many researchers through the introduction of several integral transformations [3,4]. Moreover, these transformations are studied and analyzed to discover the relationships between them [5-7]. Ali Abaoub and Abigail Shkheam recently introduced a new integral transformation, called the Abaoub-Shkheam transformation [8]. This study aims to find the dual relationships between this integral transformation and some important integral transformations, namely the Laplace transform [9], ZZ transform [10], El-Zaki transform [11], Sawi transform [12,13], Sumudu transform [14], and Mahgoub transform [15], with some simple examples.

Abaoub-Shkheam transform "Q – Transform"

Definition 1: Let (t) be a function defined for all $t \geq 0$. The Q-transform of (t) is the function defined by $f(t)$ is the function $T(v,s)$ defined by

$$Q\{f(t)\} = T(v,s) = \int_0^\infty f(vt)e^{\frac{-t}{s}}dt \qquad (1)$$

Provided the integral exists for some s, where $s\epsilon(t_1, t_2)$.
The original function $f(t)$ in eq.(1) is called the inverse transform or inverse of $T(u,s)$, and is defined by

$$f(t) = Q^{-1}[T(v,s)] = Q^{-1}\left(\int_0^\infty f(vt)e^{\frac{-t}{s}}dt\right)$$

The Abaoub-Shkheam Transform Dualities

In this section, we define the dualities between the Abaoub-Shkheam transform and some important integral transforms, namely the Laplace transform, the ZZ transform, the Elzaki transform, the Sawi Transform, the Sumudu transform, and the Mohand transform.

Abaoub-Shkheam- Laplace Duality

Definition 2: The Laplace transform of the function $f(t)$, $t \geq 0$ is given by

$$F(s) = L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt \qquad (2)$$

1Theorem

If the Laplace transform and the Abaoub-Shkheam Transform of $f(t)$ are $F(s)$ and $T(v,s)$ respectively, then

$$T(v,s) = \frac{1}{v}F\left(\frac{1}{vs}\right) \qquad (3)$$

And

$$F(s) = vT\left(v,\frac{1}{vs}\right) \qquad (4)$$

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Proof: To prove eq.(3) we use eq.(1)

$$T(v,s) = \int_0^\infty f(vt)e^{\frac{-t}{s}}dt$$

Substitute $w = vt \Rightarrow dw = vdt$ in the integral on the right-hand side, we get

$$T(v,s) = \int_0^\infty f(w)e^{\frac{-w}{vs}}\frac{dw}{v}$$

$$T(v,s) = \frac{1}{v}\int_0^\infty f(w)e^{-\left(\frac{1}{vs}\right)w}dw$$

Hence, on comparing with eq.(2) we obtain

$$T(v,s) = \frac{1}{v}F\left(\frac{1}{vs}\right)$$

To prove eq.(4), from eq.(2) above

$$\mathrm{F(s)} = \int_0^\infty f(t)e^{-st}dt$$

Substitute $t = vw \Rightarrow dt = vdw$ in the integral on right-hand side we get

$$F(s) = \int_0^\infty f(vw)e^{-svw}vdw$$

$$F(s) = v\int_0^\infty f(vw)e^{\left(-\frac{w}{\frac{1}{vs}}\right)}dw = v\int_0^\infty f(vw)e^{\left(\frac{-w}{\frac{1}{vs}}\right)}dw$$

Hence, on comparing with eq.(1) we obtain

$$F(s) = vT\left(v,\frac{1}{vs}\right)$$

Thus, we obtained the required dual relations between the Abaoub-Shkheam transform and the Laplace transform and its inverse dual.

Table 1: The dualities between the Abaoub-Shkheam transform and the Laplace transform of useful basic function

$f(t)$	$T(v,s) = Q\{f(t)\}$	$F(s) = L\{f(t)\}$	$T(v,s) = \frac{1}{v}F\left(\frac{1}{vs}\right)$
t^n	$n!\,v^n s^{n+1}$	$\frac{n!}{s^{n+1}}$	$n!\,v^n s^{n+1}$
e^{at}	$\frac{s}{1-avs}$	$\frac{1}{s-a}$	$\frac{s}{1-avs}$
$sin(at)$	$\frac{avs^2}{1+a^2v^2s^2}$	$\frac{a}{s^2+a^2}$	$\frac{avs^2}{1+a^2v^2s^2}$
$cos(at)$	$\frac{s}{1+a^2v^2s^2}$	$\frac{s}{s^2+a^2}$	$\frac{s}{1+a^2v^2s^2}$

Abaoub-Shkheam- ZZ Duality

Definition3:

Let (t) be a function defined for all $t \geq 0$. The ZZ Transform of (t) is the function $Z(v,s)$ defined by

$$Z(v,s) = G\{f(t)\} = s\int_0^\infty f(vt)e^{-st}dt \qquad (5)$$

Theorem 2

If the ZZ transform and the Abaoub-Shkheam Transform of (t) are $Z(v,s)$ and $T(v,s)$ respectively, then

$$T(v,s) = sZ\left(v,\frac{1}{s}\right) \qquad (6)$$

And

$$Z(v,s) = sT\left(v,\frac{1}{s}\right) \qquad (7)$$

Proof: To prove eq.(6), we have from eq.(1)

$$T(v,s) = \int_0^\infty f(vt)e^{\frac{-t}{s}}dt$$

Multiply $\frac{s}{s}$ in the integral on the right side, we get

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$$T(v,s)=\frac{s}{s}\int_{0}^{\infty} f(vt)e^{\frac{-t}{s}}\,dt$$

$$T(v,s)=s\left[\frac{1}{s}\int_{0}^{\infty} f(vt)e^{-\left(\frac{1}{s}\right)t}\,dt\right]$$

Hence, on comparing with eq.(5) we obtain

$$T(v,s)=sZ\left(v,\frac{1}{s}\right)$$

To prove eq.(7), from eq.(5) above

$$Z(v,s)=\ s\int_{0}^{\infty} f(vt)e^{-st}dt$$

$$Z(v,s)=s\left[\int_{0}^{\infty} f(vt)e^{\left(\frac{-t}{\frac{1}{s}}\right)}\,dt\right]$$

Hence, on comparing with eq.(1) we obtain

$$Z(v,s)=sT\left(v,\frac{1}{s}\right)$$

Thus, we obtained the required dual relations between the Abaoub-Shkheam transform and the ZZ transform and its inverse dual.

Table 2: The dualities between the Abaoub-Shkheam transform and the ZZ transform of the useful basic function

$f(t)$	$T(v,s)=Q\{f(t)\}$	$Z(v,s)=G\{f(t)\}$	$T(v,s)=sZ\left(v,\frac{1}{s}\right)$
t^n	$n!\,v^n s^{n+1}$	$\frac{n!\,v^n}{s^n}$	$n!\,v^n s^{n+1}$
e^{at}	$\frac{s}{1-avs}$	$\frac{s}{s-av}$	$\frac{s}{1-avs}$
$sin(at)$	$\frac{avs^2}{1+a^2v^2s^2}$	$\frac{avs}{s^2+a^2v^2}$	$\frac{avs^2}{1+a^2v^2s^2}$
$cos(at)$	$\frac{s}{1+a^2v^2s^2}$	$\frac{s^2}{s^2+a^2v^2}$	$\frac{s}{1+a^2v^2s^2}$

Abaoub-Shkheam- Elzaki Duality

Definition4: Elzaki transform of the function $f(t)$, $t\geq 0$ is given by

$$C(s)=\ E\{f(t)\}=\ s\int_{0}^{\infty} f(t)e^{\frac{-t}{s}}dt \qquad (8)$$

Theorem3

If the Elzaki transform and the Abaoub-Shkheam Transform of $f(t)$ are $C(s)$ and $T(v,s)$ respectively, then

$$T(v,s)=\frac{1}{sv^2}C(vs) \qquad (9)$$

And

$$C(s)=svT\left(v,\frac{s}{v}\right) \qquad (10)$$

Proof: To prove eq.(9) we use eq.(1)

$$T(v,s)=\int_{0}^{\infty} f(vt)e^{\frac{-t}{s}}dt$$

Substitute $w=vt\ \Rightarrow dw=vdt$ in the integral on the right-hand side, we get

$$T(v,s)=\int_{0}^{\infty} f(w)e^{\frac{-w}{vs}}\frac{dw}{v}$$

$$T(v,s)=\frac{1}{v}\int_{0}^{\infty} f(w)e^{-\left(\frac{1}{vs}\right)w}\,dw$$

Now multiply the right side by $\frac{vs}{vs}$ we have

$$T(v,s)=\frac{1}{sv^2}\left[vs\int_{0}^{\infty} f(w)e^{-\left(\frac{1}{vs}\right)w}\,dw\right]$$

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Hence, on comparing with eq.(8) we obtain

$$T(v,s) = \frac{1}{sv^2}C(vs)$$

To prove eq.(10), from equ.(8) above

$$C(s) = s\int_0^\infty f(t)e^{\frac{-t}{s}}dt$$

Substitute $t = vw \Rightarrow dt = vdw$ in the integral on the right-hand side, we get

$$C(s) = s\int_0^\infty f(vw)e^{\frac{-vw}{s}}vdw$$

$$C(s) = sv\int_0^\infty f(vw)e^{\left(\frac{-w}{\frac{s}{v}}\right)}dw$$

Hence, on comparing with eq.(1) we obtain

$$C(s) = svT\left(v,\frac{s}{v}\right)$$

Thus, we obtained the required dual relations between the Abaoub-Shkheam transform and the Elzaki transform and its inverse dual.

Table 3: The dualities between the Abaoub-Shkheam transform and the Elzaki Duality transform of the useful basic function

$f(t)$	$T(v,s) = Q\{f(t)\}$	$C(s) = E\{f(t)\}$	$T(v,s) = \frac{1}{sv^2}C(vs)$
t^n	$n!\,v^n s^{n+1}$	$n!\,s^{n+2}$	$n!\,v^n s^{n+1}$
e^{at}	$\frac{s}{1-avs}$	$\frac{s^2}{1-as}$	$\frac{s}{1-avs}$
$sin(at)$	$\frac{avs^2}{1+a^2v^2s^2}$	$\frac{as^3}{1+a^2s^2}$	$\frac{avs^2}{1+a^2v^2s^2}$
$cos(at)$	$\frac{s}{1+a^2v^2s^2}$	$\frac{s^2}{1+a^2s^2}$	$\frac{s}{1+a^2v^2s^2}$

Abaoub-Shkheam- Sawi Duality

Definition 5: Sawi transforms of the function $f(t)$, $t \geq 0$ is defined by

$$A(s) = W\{f(t)\} = \frac{1}{s^2}\int_0^\infty f(t)e^{\frac{-t}{s}}dt \qquad (11)$$

Theorem 4

If the Sawi transforms and the Abaoub-Shkheam Transform of (t) are $A(s)$ and $T(v,s)$ respectively, then

$$T(v,s) = vs^2A(vs) \qquad (12)$$

And

$$A(s) = \frac{v}{s^2}T\left(v,\frac{s}{v}\right) \qquad (13)$$

Proof: To prove eq.(12), we have from eq.(1)

$$T(v,s) = \int_0^\infty f(vt)e^{\frac{-t}{s}}dt$$

Substitute $w = vt \Rightarrow dw = vdt$ in the integral on the right-hand side, we get

$$T(v,s) = \int_0^\infty f(w)e^{\frac{-w}{vs}}\frac{dw}{v}$$

$$T(v,s) = \frac{1}{v}\int_0^\infty f(w)e^{-\left(\frac{1}{vs}\right)w}dw$$

Now multiply the right side by $\frac{vs^2}{vs^2}$ we have

$$T(v,s) = \frac{vs^2}{vs^2}\cdot\frac{1}{v}\int_0^\infty f(w)e^{-\left(\frac{1}{vs}\right)w}dw = vs^2\left[\frac{1}{(vs)^2}\int_0^\infty f(w)e^{-\left(\frac{1}{vs}\right)w}dw\right]$$

Hence, on comparing with eq.(11) we obtain

$$T(v,s) = vs^2A(vs)$$

To prove eq.(13), from eq.(11) above

$$A(s) = \frac{1}{s^2}\int_0^\infty f(t)e^{\frac{-t}{s}}dt$$

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Substitute $t = vw \Rightarrow dt = vdw$ in the integral on the right-hand side, we get

$$A(s) = \frac{1}{s^2} \cdot v \int_0^\infty f(vw) e^{-\frac{vw}{s}} dw$$

$$A(s) = \frac{v}{s^2} \int_0^\infty f(vw) e^{-\left(\frac{w}{\frac{s}{v}}\right)} dw$$

Hence, on comparing with eq. (1) we obtain

$$A(s) = \frac{v}{s^2} T\left(v, \frac{s}{v}\right)$$

Thus, we obtained the required dual relations between the Abaoub-Shkheam transform and the Sawi transform and its inverse dual.

Table 4: The dualities between the Abaoub-Shkheam transform and Sawi transform of the useful basic function

$f(t)$	$T(v,s) = Q\{f(t)$	$A(s) = W\{f(t)$	$T(v,s) = vs^2 A(vs)$
t^n	$n!\, v^n s^{n+1}$	$n!\, s^{n-1}$	$n!\, v^n s^{n+1}$
e^{at}	$\frac{s}{1-avs}$	$\frac{1}{s(1-as)}$	$\frac{s}{1-avs}$
$sin(at)$	$\frac{avs^2}{1+a^2v^2s^2}$	$\frac{a}{1+a^2s^2}$	$\frac{avs^2}{1+a^2v^2s^2}$
$cos(at)$	$\frac{s}{1+a^2v^2s^2}$	$\frac{1}{s(1+a^2s^2)}$	$\frac{s}{1+a^2v^2s^2}$

Abaoub-Shkheam- Sumudu Duality

Definition 6: Sumudu transform of the function $f(t)$, $t \geq 0$ is given by

$$J(s) = U\{f(t)\} = \int_0^\infty f(st) e^{-t} dt \qquad (14)$$

Theorem5

If the Sumudu transform and the Abaoub-Shkheam Transform of $f(t)$ are $J(s)$ and $T(v,s)$ respectively, then

$$T(v,s) = sJ(vs) \qquad (15)$$

And

$$J(s) = \frac{v}{s} T\left(v, \frac{s}{v}\right) \qquad (16)$$

Proof: To prove eq.(15) we use eq.(1)

$$T(v,s) = \int_0^\infty f(vt) e^{\frac{-t}{s}} dt$$

Substitute $w = \frac{t}{s} \Rightarrow dw = \frac{dt}{s}$ in the integral on the right-hand side, we get

$$T(v,s) = \int_0^\infty f(vsw) e^{-w}\, s dw$$

$$T(v,s) = s \int_0^\infty f((vs)w) e^{-w} dw$$

Hence, on comparing with eq.(14) we obtain

$$T(v,s) = sJ(vs)$$

To prove eq.(16), from eq.(14) above

$$\mathrm{J(s)} = \int_0^\infty f(st) e^{-t} dt$$

Substitute $t = \frac{vw}{s} \Rightarrow dt = \frac{v}{s} dw$ in the integral on the right-hand side, we get

$$J(s) = \int_0^\infty f(vw) e^{\frac{-vw}{s}} \frac{v}{s} dw$$

$$F(s) = \frac{v}{s} \int_0^\infty f(vw) e^{\left(-\frac{w}{\frac{s}{v}}\right)} dw$$

Hence, on comparing with eq.(1) we obtain

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$$J(s) = \frac{v}{s} T\left(v, \frac{s}{v}\right)$$

Thus, we obtained the required dual relations between the Abaoub-Shkheam transform and the Sumudu transform and its inverse dual.

Table 5: The dualities between the Abaoub-Shkheam transform and the Sumudu transform of the useful basic function

$f(t)$	$T(v,s) = Q\{f(t)\}$	$J(s) = U\{f(t)\}$	$T(v,s) = sJ(vs)$
t^n	$n!\, v^n s^{n+1}$	$n!\ s^n$	$n!\, v^n s^{n+1}$
e^{at}	$\frac{s}{1-avs}$	$\frac{1}{1-as}$	$\frac{s}{1-avs}$
$sin(at)$	$\frac{avs^2}{1+a^2v^2s^2}$	$\frac{a\,s}{1+a^2s^2}$	$\frac{avs^2}{1+a^2v^2s^2}$
$cos(at)$	$\frac{s}{1+a^2v^2s^2}$	$\frac{1}{1+a^2s^2}$	$\frac{s}{1+a^2v^2s^2}$

Abaoub-Shkheam -Mahgoub Duality

Definition 7:Let $f(t)$ be a function defined for all $t \geq 0$. The Mahgoub Transform of (t) is the function $H(s)$ defined by

$$H(s) = M\{f(t)\} = s\int_0^\infty f(t)e^{-st}dt \qquad (17)$$

Theorem 6

If the Mahgoub transform and the Abaoub-Shkheam Transform of $f(t)$ are $H(s)$ and $T(v,s)$ respectively, then

$$T(v,s) = sH\left(\frac{1}{vs}\right) \qquad (18)$$

And

$$H(s) = vsT\left(v, \frac{1}{vs}\right) \qquad (19)$$

Proof: To prove eq.(18) we use eq.(1)

$$T(v,s) = \int_0^\infty f(vt)e^{\frac{-t}{s}}dt$$

Substitute $w = vt \Rightarrow dw = vdt$ in the integral on the right-hand side, we get

$$T(v,s) = \int_0^\infty f(w)e^{\frac{-w}{vs}}\frac{dw}{v}$$

$$T(v,s) = \frac{1}{v}\int_0^\infty f(w)e^{-\left(\frac{1}{vs}\right)w}\,dw$$

Multiply $\frac{s}{s}$ in the integral on the right-hand side, we get

$$T(v,s) = s\left[\frac{1}{vs}\int_0^\infty f(w)e^{-\left(\frac{1}{vs}\right)w}\,dw\right]$$

Hence, on comparing with eq.(17) we obtain

$$T(v,s) = sH\left(\frac{1}{vs}\right)$$

To prove eq.(19), from eq.(17) above

$$H(s) = s\int_0^\infty f(t)e^{-st}\,dt$$

Substitute $t = vw \Rightarrow dt = vdw$ in the integral on the right-hand side, we get

$$H(s) = s\int_0^\infty f(vw)e^{-svw}\,vdw$$

$$H(s) = vs\int_0^\infty f(vw)e^{\left(\frac{-w}{\frac{1}{vs}}\right)}\,dw$$

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Hence, on comparing with eq.(1) we obtain

$$H(s) = vsT\left(v, \frac{1}{vs}\right)$$

Thus, we obtained the required dual relations between the Abaoub-Shkheam transform and the Mahgoub transform and its inverse dual.

Table 6: The dualities between the Abaoub-Shkheam transform and the Mahgoub transform of the useful basic function

$f(t)$	$T(v,s) = Q\{f(t)$	$H(s) = M\{f(t)\}$	$T(v,s) = sH\left(\frac{1}{vs}\right)$
t^n	$n!\,v^n s^{n+1}$	$\frac{n!}{s^n}$	$n!\,v^n s^{n+1}$
e^{at}	$\frac{s}{1-avs}$	$\frac{s}{s-a}$	$\frac{s}{1-avs}$
$sin(at)$	$\frac{avs^2}{1+a^2v^2s^2}$	$\frac{a\,s}{s^2+a^2}$	$\frac{avs^2}{1+a^2v^2s^2}$
$cos(at)$	$\frac{s}{1+a^2v^2s^2}$	$\frac{s^2}{s^2+a^2}$	$\frac{s}{1+a^2v^2s^2}$

Examples

Example 1: Consider the function $f(t) = t + sint$ Abaoub-Shkheam, ZZ, and Elzaki transforms are obtained respectively as follows:

$$T(v,s) = Q\{f(t)\} = vs^2 + \frac{vs^2}{1+v^2s^2}$$

From eq.(7) we get

$$Z(v,s) = sT\left(v, \frac{1}{s}\right) = \frac{v}{s} + \frac{vs}{v^2+s^2}$$

And from eq.(10) we get

$$C(s) = svT\left(v, \frac{s}{v}\right) = s^3 + \frac{s^3}{1+s^2}$$

Example 2: Consider the function $f(t) = t^2e^t$ Laplace, Abaoub-Shkheam, and Mahgoub transforms are obtained, respectively, as follows:

$$F(s) = L\{f(t)\} = \frac{2}{(s-1)^3}$$

From eq.(3) we get

$$T(v,s) = \frac{1}{v}F\left(\frac{1}{vs}\right) = \frac{2v^2s^3}{(1-vs)^3}$$

And from eq.(19) we get

$$H(s) = vsT\left(v, \frac{1}{vs}\right) = \frac{2s}{(s-1)^3}$$

Conclusion

In this paper, we have discussed the dual relations between the Abaoub-Shkheam integral transform and the Laplace, ZZ, Elzaki, Sawi, Sumudu, and Mahgoub integral transforms, as well as their inverse dualities. We have also used tabular presentation of the integral transforms of the most used fundamental functions to show, using this, the dual relations, to illustrate the importance of these dualities between the integral transforms mentioned above and the Abaoub-Shkheam transform and their interrelationship, and supported with some simple examples. In the future, using these dual relations, we can easily solve many advanced problems of the modern era, such as the motion of coupled harmonic oscillators, drug distribution in the body, and common health problems, such as the detection of diabetes and tumor growth.

Conflict of interest. Nil

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