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Original article

Stability Analysis of Malaria Transmission Model, Deterministic and Stochastic using Lyapunov Functions and Numerical Methods (Euler and Euler-Maruyama)

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Abstract
Malaria remains one of the most life-threatening infectious diseases worldwide, causing hundreds of thousands of deaths annually, particularly among children and pregnant women in tropical and subtropical regions. To better understand its complex transmission dynamics, this study develops and analyzes both deterministic and stochastic mathematical models of malaria based on systems of differential equations. The main objective of this research is to study the stability of both the deterministic and stochastic malaria models analytically and numerically using the Lyapunov function, the Euler method, and the Euler–Maruyama method. For the deterministic model, the disease-free and endemic equilibrium points are derived, and their local and global stability are investigated. Numerical simulations are conducted using the mentioned numerical methods to demonstrate the dynamic behavior of the system. The stochastic extension of the model incorporates random perturbations to represent environmental and demographic fluctuations that influence disease spread. The analytical and numerical results reveal that stochastic effects significantly influence malaria dynamics, potentially reducing disease persistence and stabilizing the system under certain conditions. These findings provide deeper insights into malaria transmission mechanisms and contribute to the development of more effective and evidence-based control strategies.
Keywords. Malaria Transmission Model, Deterministic Model, Lyapunov Function, Stochastic Model.

Introduction

Malaria presents a significant global public health challenge, responsible for hundreds of thousands of deaths annually. It is particularly noted that young children are most at risk, accounting for the majority of these fatalities. Studies indicate that acquired immunity to the disease develops with repeated exposure to parasites, making it a crucial factor influencing disease transmission dynamics. Therefore, it is essential to develop precise mathematical models to analyze these dynamics and understand the complex interplay between levels of disease exposure, the mechanism of immunity acquisition, and the infection pathway [1]. Furthermore, malaria is a parasitic infection that is transmitted to humans by the bite of specific mosquito species known as anophelines. The parasite must spend a portion of its existence in the mosquito, rather than simply moving from one person to another.

Because the parasite's growth and development take almost as long as the insect's typical lifespan, life within the mosquito is a race against time. In colder climates, this time frame is longer; as the temperature increases, it gets shorter. Therefore, the parasite's existence is precarious, and the mosquito usually dies before it can spread malaria once the average temperature falls below a particular threshold. This explains why malaria poses such a serious risk to health in tropical and subtropical areas [2]. In many regions of the world, malaria is endemic, but it is most prevalent in sub-Saharan Africa. The World Health Organization (WHO) estimates that 229 million cases of malaria occurred globally in 2019, resulting in about 409,000 fatalities. A startling 94% of all instances and fatalities occurred in Africa, with pregnant women and children under five being the most susceptible groups [3].

Mathematical models play a crucial role in the study of malaria transmission dynamics, having been utilized for several decades to provide a comprehensive framework for understanding the factors influencing the spread of this disease. Initial efforts in this area employed simple models based on ordinary differential equations, which helped elucidate changes in the densities of infected humans and mosquitoes. As research has progressed, these models have evolved to incorporate greater complexities, such as the latent periods during which individuals are infected but asymptomatic, as well as the impact of vector density and the age structure of populations. Additionally, other factors such as migration, social and economic changes, and climatic influences have been integrated, thereby enhancing the accuracy of the modeling. These models highlight the intricacies present in the interactions among hosts, vectors, and parasites, underscoring the need for models that account for environmental and social diversity.

Through this comprehensive approach, mathematical models can provide valuable insights into the potential risks of malaria transmission, facilitating the development of effective strategies for disease control [4].

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Among these, deterministic models serve as a pivotal analytical tool in the study of complex biological systems, offering a systematic framework for understanding the interactions and dynamics of infectious diseases. By employing systems of differential equations, these models enable the precise modeling of disease transmission pathways and the identification of influential factors on its spread. The fundamental role of deterministic models lies in their ability to isolate different variables and assess their impact independently, which facilitates sensitivity analyses to pinpoint the most critical parameters in controlling disease dynamics. Furthermore, these models provide quantifiable insights that can effectively guide the formulation of intervention and prevention strategies, making them an indispensable tool in the field of mathematical epidemiology [5].

However, stochastic modeling holds paramount importance in epidemiology, offering an analytical framework that surpasses the limitations of traditional deterministic models, particularly when studying epidemics in small populations. While deterministic models presuppose a single, predictable trajectory for disease progression, stochastic models inherently incorporate an element of randomness. This inherent stochasticity, whether demographic (e.g., variations in transmission or recovery rates at the individual level) or environmental (e.g., changes in external factors), is fundamental to shaping epidemic dynamics. Consequently, these models allow for the estimation of the probability of events (such as an outbreak) rather than merely predicting their definitive occurrence. Their ability to capture the cumulative effect of small random events makes them an indispensable tool for forecasting the trajectories of diseases influenced by individual or environmental variability, thereby enhancing the accuracy of predictions and providing a more comprehensive view of complex epidemiological phenomena [6].

A basic mathematical framework for examining the stability of dynamical systems is provided by the use of Lyapunov functions. This approach becomes particularly significant when applied to mathematical models of infectious diseases, as it enables the assessment of their long-term behaviors and the stability of their equilibrium states [7]. In the context of analyzing epidemiological models, the Lyapunov function represents a core mathematical tool for verifying the stability of systems and the accuracy of mathematical models. One recent study utilized a stochastic Lyapunov functional analysis to evaluate a fractional model describing the co-dynamics of both malaria and COVID-19. The study's results showed that this approach not only proves the global stability of the system but also confirms the existence of a unique solution, which enhances the model's reliability and provides a solid foundation for its use in biological and epidemiological applications [8].

Building on this, the current study aims to use Lyapunov functions to analyze the stability of a stochastic differential equations model for malaria transmission. Furthermore, the study seeks to apply numerical methods such as Euler method and Euler–Maruyama method to simulate the model and obtain accurate approximate solutions, thereby providing new insights into disease dynamics and contributing to the development of more effective and evidence-based control strategies [8].

The remainder of this paper is organized as follows: Section 2 presents the formulation of the malaria transmission model. Section 3 discusses the disease-free and endemic equilibria and derives the basic reproduction number, and analyzes stability using the Lyapunov function and Euler method. Section 4 analyzes the stochastic version of the model and its stability using Lyapunov functions. Section 5 provides numerical simulations based on the Euler–Maruyama method, and Section 6 concludes with the main findings and implications. Based on the above discussion, the next section introduces the mathematical formulation of the malaria transmission model that forms the foundation of our analysis.

Model formulation

To analyze the dynamics of malaria spread and understand the impact of different parameters on the disease outbreak, a mathematical model based on differential equations was developed. This model allows studying changes in the number of infected and susceptible individuals over time.

In this section, the stability of the system will be studied by calculating the basic reproduction number R_0 and using the Lyapunov function and numerical analysis to evaluate the effect of different parameters on the spread of the disease.

Here we apply the SEIR-SEI of Malaria. The model is formulated for both the human population as well as mosquito population at time t, For humans, we divide our population into four classes: Susceptible S_H, Exposed E_H, Infectious I_H, and Recovery Human R_H. And the population of the mosquitoes is divided into three classes, namely Susceptible S_V, Exposed E_V, and Infectious I_V.

The following model shows the path of infection between humans and mosquitoes.

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$$\begin{cases} \frac{dS_H}{dt} = \Lambda_H - \beta_H S_H I_H - \mu_H S_H + \rho R_H \\ \frac{dE_H}{dt} = \beta_H S_H I_H - (\alpha_1 + \mu_H) E_H \\ \frac{dI_H}{dt} = \alpha_1 E_H - (\alpha_2 + \mu_H + \delta) I_H + \psi I_H \\ \frac{dR_H}{dt} = \alpha_2 I_H - (\mu_H + \rho) R_H \\ \frac{dS_V}{dt} = \Lambda_V - \beta_V S_V I_V - \mu_V S_V \\ \frac{dE_V}{dt} = \beta_V S_V I_V - (\alpha_3 + \mu_V) E_V \\ \frac{dI_V}{dt} = \alpha_3 E_V - \mu_V I_V \end{cases} . \quad (1)$$

Table 1. List of the parameters. *[9].*

Parameter	Description
Λ_H.	Recruitment rate of humans.
Λ_V.	Recruitment rate of mosquitoes.
δ.	Induce the death rate of humans.
ρ.	Loss of immunity for humans.
ψ.	The rate of newborns' birth with infection humans.
α_1.	The developing rate of exposed (humans) becoming infectious.
α_2.	Recovery rate of humans (removal rate).
α_3.	The developing rate of exposed mosquitoes becoming infectious.
μ_H.	Natural death rate of humans.
μ_V.	Natural death rate of mosquitoes.
q_H.	Probability of transmission of infection from an infectious mosquito to a susceptible human.
q_V.	Probability of transmission of infection from an infectious human to a susceptible mosquito.
η_V.	Mosquitoes biting rate.
β_H.	Infection rate $q_H \times \eta_V$ of humans.
β_V.	Infection rate $q_V \times \eta_V$ mosquitoes.

Where: $S_H(t) = S_H(0), E_H(t) = E_H(0), I_H(t) = I_H(0), R_H(t) = R_H(0), S_V(t) = S_V(0), E_V(t) = E_V(0), I_V(t) = I_V(0)$.
Since all model parameters represent biologically meaningful rates such as infection weight equations, progress equations in recovery rates, natural death rates, and death rates resulting from disease, all of these parameters are positive.

Model analysis

Disease-Free Equilibrium (DFE):

If $I_H = E_H = R_H = I_V = E_V = 0$.

Then $\Lambda_H - \beta_H S_H I_H - \mu_H S_H + \rho R_H = 0 \Longrightarrow \Lambda_H - \mu_H S_H = 0 \Longrightarrow S_H^0 = \frac{\Lambda_H}{\mu_H}$,

And $\Lambda_V - \beta_V S_V I_V - \mu_V S_V = 0 \Longrightarrow \Lambda_V - \mu_V S_V = 0 \Longrightarrow S_V^0 = \frac{\Lambda_V}{\mu_V}$.

Hence, the DFE is $\{S_H^0, E_H^0, I_H^0, R_H^0, S_V^0, E_V^0, I_V^0\} = \left(\frac{\Lambda_H}{\mu_H}, 0,0,0, \frac{\Lambda_V}{\mu_V}, 0,0\right)$.

After determining the disease-free point, we now move on to calculating the basic reproduction number R_0, which is used to analyze the stability of this point.

Basic Reproduction Number R_0.

To calculate R_0 using the next generation matrix methodology, we first divide the variables into infectious and non-infectious categories as follows:

Let X be the vector of infectious cases and Y be the vector of non-infectious cases, such that:

$X = (E_H, I_H, E_V, I_V)^T, Y = (S_H, R_H, S_V)^T$.

$$F(X) = \begin{pmatrix} \beta_H S_H I_H \\ 0 \\ \beta_V S_V I_V \\ 0 \end{pmatrix}, M(X) = \begin{pmatrix} -(\alpha_1 + \mu_H) E_H \\ (\alpha_2 + \mu_H + \delta) I_H - \alpha_1 E_H - \psi I_H \\ -(\alpha_3 + \mu_V) E_V \\ \mu_V I_V - \alpha_3 E_V \end{pmatrix}.$$

The vector $F(X)$ represents the rates of new infections, while $M(X)$ describes the rates of transmission.

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$$J(F(X)) = \begin{pmatrix} \frac{\partial F_1}{\partial E_H} & \frac{\partial F_1}{\partial I_H} & \frac{\partial F_1}{\partial E_V} & \frac{\partial F_1}{\partial I_V} \\ \frac{\partial F_2}{\partial E_H} & \frac{\partial F_2}{\partial I_H} & \frac{\partial F_2}{\partial E_V} & \frac{\partial F_2}{\partial I_V} \\ \frac{\partial F_3}{\partial E_H} & \frac{\partial F_3}{\partial I_H} & \frac{\partial F_3}{\partial E_V} & \frac{\partial F_3}{\partial I_V} \\ \frac{\partial F_4}{\partial E_H} & \frac{\partial F_4}{\partial I_H} & \frac{\partial F_4}{\partial E_V} & \frac{\partial F_4}{\partial I_V} \end{pmatrix}, and\ J(M(X)) = \begin{pmatrix} \frac{\partial V_1}{\partial E_H} & \frac{\partial V_1}{\partial I_H} & \frac{\partial V_1}{\partial E_V} & \frac{\partial V_1}{\partial I_V} \\ \frac{\partial V_2}{\partial E_H} & \frac{\partial V_2}{\partial I_H} & \frac{\partial V_2}{\partial E_V} & \frac{\partial V_2}{\partial I_V} \\ \frac{\partial V_3}{\partial E_H} & \frac{\partial V_3}{\partial I_H} & \frac{\partial V_3}{\partial E_V} & \frac{\partial V_3}{\partial I_V} \\ \frac{\partial V_4}{\partial E_H} & \frac{\partial V_4}{\partial I_H} & \frac{\partial V_4}{\partial E_V} & \frac{\partial V_4}{\partial I_V} \end{pmatrix}.$$

By taking the partial derivatives of the vectors $F(X)$ and $M(X)$ with respect to the contagious variables, we obtain two Jacobian matrices $J(F(X))\ and\ J(M(X))$:

$$\text{Then } J(F(X)) = \begin{pmatrix} 0 & \beta_H S_H & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_V S_V \\ 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\text{and } J(M(X)) = \begin{pmatrix} (\alpha_1 + \mu_H) & 0 & 0 & 0 \\ -\alpha_1 & (\alpha_2 + \mu_H + \delta) - \psi & 0 & 0 \\ 0 & 0 & (\alpha_3 + \mu_V) & 0 \\ 0 & 0 & -\alpha_3 & \mu_V \end{pmatrix}.$$

$$J(F(DFE)) = \begin{pmatrix} 0 & \frac{\beta_H \Lambda_H}{\mu_H} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_V \Lambda_V}{\mu_V} \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\text{and } J(M(DFE)) = \begin{pmatrix} (\alpha_1 + \mu_H) & 0 & 0 & 0 \\ -\alpha_1 & (\alpha_2 + \mu_H + \delta) - \psi & 0 & 0 \\ 0 & 0 & (\alpha_3 + \mu_V) & 0 \\ 0 & 0 & -\alpha_3 & \mu_V \end{pmatrix}.$$

From the Jacobian matrix of $M(X)$ at the disease-free state (DFE), we note that the matrix takes a (block-diagonal) form, separating the dynamics of humans from the dynamics of mosquitoes (E_V, I_V). Since there are no connecting elements between the two masses, we can easily calculate the inverse of M by inverting each mass separately.[10]

$$J(M(DFE)) = M = \begin{pmatrix} M_H & 0 \\ 0 & M_V \end{pmatrix} \Rightarrow M^{-1} = \begin{pmatrix} M_H^{-1} & 0 \\ 0 & M_V^{-1} \end{pmatrix}.$$

$$M_H = \begin{pmatrix} (\alpha_1 + \mu_H) & 0 \\ -\alpha_1 & (\alpha_2 + \mu_H + \delta) - \psi \end{pmatrix} \text{ and } M_V = \begin{pmatrix} (\alpha_3 + \mu_V) & 0 \\ -\alpha_3 & \mu_V \end{pmatrix}.$$

$$M_H^{-1} = \frac{1}{\det(V_H)} adj\ (V_H).$$

$$= \frac{1}{(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} \begin{pmatrix} (\alpha_2 + \mu_H + \delta - \psi) & 0 \\ \alpha_1 & (\alpha_1 + \mu_H) \end{pmatrix}.$$

$$M_H^{-1} = \begin{pmatrix} \frac{1}{(\alpha_1+\mu_H)} & 0 \\ \frac{\alpha_1}{(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} & \frac{1}{(\alpha_2+\mu_H+\delta-\psi)} \end{pmatrix}.$$

$$M_V^{-1} = \frac{1}{\det(V_V)} adj\ (V_V).$$

$$= \frac{1}{(\alpha_3+\mu_V)\mu_V} \begin{pmatrix} \mu_V & 0 \\ \alpha_3 & (\alpha_3 + \mu_V) \end{pmatrix}.$$

$$M_H^{-1} = \begin{pmatrix} \frac{1}{(\alpha_3+\mu_V)} & 0 \\ \frac{\alpha_3}{(\alpha_3+\mu_V)\mu_V} & \frac{1}{\mu_V} \end{pmatrix}.$$

$$\therefore M^{-1} = \begin{pmatrix} \frac{1}{(\alpha_1+\mu_H)} & 0 & 0 & 0 \\ \frac{\alpha_1}{(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} & \frac{1}{(\alpha_2+\mu_H+\delta-\psi)} & 0 & 0 \\ 0 & 0 & \frac{1}{(\alpha_3+\mu_V)} & 0 \\ 0 & 0 & \frac{\alpha_3}{(\alpha_3+\mu_V)\mu_V} & \frac{1}{\mu_V} \end{pmatrix}.$$

Thus, we get the next generation matrix $K = F.M^{-1}$ and the maximum eigenvalue of this matrix is given by the basic multiplication number R_0.

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$$K = \begin{pmatrix} 0 & \frac{\beta_H \Lambda_H}{\mu_H} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta_V \Lambda_V}{\mu_V} \\ 0 & 0 & 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{(\alpha_1+\mu_H)} & 0 & 0 & 0 \\ \frac{\alpha_1}{(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} & \frac{1}{(\alpha_2+\mu_H+\delta-\psi)} & 0 & 0 \\ 0 & 0 & \frac{1}{(\alpha_3+\mu_V)} & 0 \\ 0 & 0 & \frac{\alpha_3}{(\alpha_3+\mu_V)\mu_V} & \frac{1}{\mu_V} \end{pmatrix}.$$

$$K = \begin{pmatrix} \frac{\alpha_1(\beta_H \Lambda_H)}{\mu_H(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} & \frac{\beta_H \Lambda_H}{\mu_H(\alpha_2+\mu_H+\delta-\psi)} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha_3(\beta_V \Lambda_V)}{(\alpha_3+\mu_V)(\mu_V)^2} & \frac{\beta_V \Lambda_V}{(\mu_V)^2} \\ 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$K_H = \begin{pmatrix} \frac{\alpha_1(\beta_H \Lambda_H)}{\mu_H(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} & \frac{\beta_H \Lambda_H}{\mu_H(\alpha_2+\mu_H+\delta-\psi)} \\ 0 & 0 \end{pmatrix}.$$

and $K_V = \begin{pmatrix} \frac{\alpha_3(\beta_V \Lambda_V)}{(\alpha_3+\mu_V)(\mu_V)^2} & \frac{\beta_V \Lambda_V}{(\mu_V)^2} \\ 0 & 0 \end{pmatrix}$.

$$|K_H - \lambda I| = 0.$$

$$\left| \begin{pmatrix} \frac{\alpha_1(\beta_H \Lambda_H)}{\mu_H(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} & \frac{\beta_H \Lambda_H}{\mu_H(\alpha_2+\mu_H+\delta-\psi)} \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0.$$

$$\left| \begin{matrix} \frac{\alpha_1(\beta_H \Lambda_H)}{\mu_H(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} - \lambda & \frac{\beta_H \Lambda_H}{\mu_H(\alpha_2+\mu_H+\delta-\psi)} \\ 0 & -\lambda \end{matrix} \right| = 0.$$

$$\left(\frac{\alpha_1(\beta_H \Lambda_H)}{\mu_H(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)}\right)\lambda - \lambda^2 = 0 \Longrightarrow \lambda_1 = 0 \; and \; \lambda_2 = \frac{\alpha_1(\beta_H \Lambda_H)}{\mu_H(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)}.$$

$$\therefore R_{0,H} = \frac{\alpha_1(\beta_H \Lambda_H)}{\mu_H(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)}.$$

$$|K_V - \lambda I| = 0.$$

$$\left| \begin{pmatrix} \frac{\alpha_3(\beta_V \Lambda_V)}{(\alpha_3+\mu_V)(\mu_V)^2} & \frac{\beta_V \Lambda_V}{(\mu_V)^2} \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0.$$

$$\left| \begin{matrix} \frac{\alpha_3(\beta_V \Lambda_V)}{(\alpha_3+\mu_V)(\mu_V)^2} - \lambda & \frac{\beta_V \Lambda_V}{(\mu_V)^2} \\ 0 & -\lambda \end{matrix} \right| = 0.$$

$$\left(\frac{\alpha_3(\beta_V \Lambda_V)}{(\alpha_3+\mu_V)(\mu_V)^2}\right)\lambda - \lambda^2 = 0 \Longrightarrow \lambda_1 = 0 \; and \; \lambda_2 = \frac{\alpha_3(\beta_V \Lambda_V)}{(\alpha_3+\mu_V)(\mu_V)^2}.$$

$$\therefore R_{0,V} = \frac{\alpha_3(\beta_V \Lambda_V)}{(\alpha_3+\mu_V)(\mu_V)^2}.$$

Hence $R_0 = \sqrt{R_{0,H}, R_{0,V}}$.

In infection models such as malaria, the infection passes through two species, humans and mosquitoes, so the basic reproduction number for one cycle is the square root of the average infection rate between the two species.

$$R_0 = \sqrt{\frac{\alpha_1(\beta_H \Lambda_H)}{\mu_H(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} \cdot \frac{\alpha_3(\beta_V \Lambda_V)}{(\alpha_3+\mu_V)(\mu_V)^2}} = \sqrt{\frac{\alpha_1(\beta_H S_H^0)}{(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi)} \cdot \frac{\alpha_3(\beta_V S_V^0)}{(\alpha_3+\mu_V)\mu_V}} = \sqrt{\frac{\alpha_1(\beta_H S_H^0).\alpha_3(\beta_V S_V^0)}{(\alpha_1+\mu_H)(\alpha_2+\mu_H+\delta-\psi).(\alpha_3+\mu_V)\mu_V}}.$$

1- If $R_0 \leq 1$, the disease-free equilibrium point is stable, and disease disappears from the population.

2- If $R_0 > 1$, the disease-free equilibrium point is unstable, allowing the disease to emerge and spread in the community.

Due to the complexity of the analytical expression for R_0 and the inability to determine its sign conclusively, it was not possible to prove the local stability of the disease-free point. Therefore, we adopted the Lyapunov function to prove the global stability of the system, which gives an accurate result independent of the values of R_0.

Stability analysis using the Lyapunov function of DFE:

We first divide the variables into two groups:

Human variables are $X = \{S_H, E_H, I_H, R_H\}$.

Variables specific to the mosquitoes is $Y = \{S_V, E_V, I_V\}$.

According to the methodology of (Korobeinikov) and colleagues, we divide the system into two parts: the human system $F(X)$ and the carrier system (the mosquito) $F(Y)$. For each part a function is volterra based on the combination of linear and nonlinear terms [11].

$$F(X) = \begin{cases} \frac{dS_H}{dt} = \Lambda_H - \beta_H S_H I_H - \mu_H S_H + \rho R_H \\ \frac{dE_H}{dt} = \beta_H S_H I_H - (\alpha_1 + \mu_H) E_H \\ \frac{dI_H}{dt} = \alpha_1 E_H - (\alpha_2 + \mu_H + \delta) I_H + \psi I_H \\ \frac{dR_H}{dt} = \alpha_2 I_H - (\mu_H + \rho) R_H \end{cases}, \qquad F(Y) = \begin{cases} \frac{dS_V}{dt} = \Lambda_V - \beta_V S_V I_V - \mu_V S_V \\ \frac{dE_V}{dt} = \beta_V S_V I_V - (\alpha_3 + \mu_V) E_V \\ \frac{dI_V}{dt} = \alpha_3 E_V - \mu_V I_V \end{cases}.$$

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The general form of the Volterra function can be expressed as follows:

$$U = \sum_i \alpha_i \left(x_i - x_i^* - x_i^* ln\frac{x_i}{x_i^*}\right) s.t\ x_i - x_i^* - x_i^* ln\frac{x_i}{x_i^*} > 0\ \forall x_i > 0\ .$$

This type of Volterra function has been used because of its suitability for global stability analysis in epidemiological models, especially in cases where the value of R_0 is not defined as being greater or less than 1 [12-13].

In many previous studies, the shape of the Lyapunov function is chosen according to the value of R_0, where simple linear or quadratic functions are used when $R_0 \leq 1$, while different functions are constructed when $R_0 > 1$ to prove endemic stability [14].

Because the Volterra function is logarithmic in form, which guarantees its positivity for all positive values of the variables, this formulation makes the derivative of the function negative or zero only at equilibrium, which confirms its effectiveness in analyzing the stability of the system directly without the need to solve differential equations explicitly. In this way, the classical theory of Volterra functions is linked to the stability analysis of traditional epidemiological models, providing a powerful tool for assessing the dynamics of infectious diseases.[13]

To understand how to achieve the conditions for practical Lyapunov, the work methodology was explained step by step. This reflects the validity and soundness of the presented analysis and confirms the reliability of the results obtained.

1- The Volterra function of $F(X)$ is

$$U_1(X) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right) + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right) + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right) > 0\ \forall\ S_H, E_H, I_H > 0\ .$$

The R_H class was not included in the Volterra function because it does not directly contribute to the transmission dynamics and can be represented in terms of other variables through the population conservation relationship. Therefore, omitting it does not affect the results of the stability analysis. Rather, it simplifies the function and preserves its analytical effectiveness.

2- The Volterra function of $F(Y)$ is

$$U_2(Y) = \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right) + \alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right) + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right) > 0\ \forall\ S_V, E_V, I_V > 0.$$

Proving the positivity of weights in the Lyapunov function:

Since the variable representing S_H healthy individuals is not a source of direct infection, we assumed that its weight in the Lyapunov function is equal to one $\alpha_{S_H} = 1$.

Now $\because \alpha_{S_H} = 1$, then $\alpha_{E_H}(\beta_H S_H I_H) = \alpha_{S_H}(\beta_H S_H I_H) \Longrightarrow \alpha_{E_H}(\beta_H S_H I_H) = (1)(\beta_H S_H I_H) \Longrightarrow \alpha_{E_H} = 1$.

$\alpha_{I_H}(\alpha_1 E_H) = \alpha_{E_H}(\beta_H S_H I_H) \Longrightarrow \alpha_{I_H}(\alpha_1 E_H) = (1)(\beta_H S_H I_H) \Longrightarrow \alpha_{I_H} = \frac{(\beta_H S_H I_H)}{(\alpha_1 E_H)}$.

$\alpha_{S_V}(\beta_V S_V I_V) = \alpha_{I_H}(\alpha_1 E_H) \Longrightarrow \alpha_{S_V}(\beta_V S_V I_V) = (\frac{(\beta_H S_H I_H)}{(\alpha_1 E_H)})(\alpha_1 E_H) \Longrightarrow \alpha_{S_V} = \frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}$.

$\alpha_{E_V}(\beta_V S_V I_V) = \alpha_{S_V}(\beta_V S_V I_V) \Longrightarrow \alpha_{E_V}(\beta_V S_V I_V) = (\frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)})(\beta_V S_V I_V) \Longrightarrow \alpha_{E_V} = \frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}$.

$\alpha_{I_V}(\alpha_3 E_V) = \alpha_{E_V}(\beta_V S_V I_V) \Longrightarrow \alpha_{I_V}(\alpha_3 E_V) = \frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}(\beta_V S_V I_V)) \Longrightarrow \alpha_{I_V} = \frac{(\beta_H S_H I_H)}{(\alpha_3 E_V)}$.

These weights were determined from the limits responsible for transmission of infection in the system, i.e. those that link infected and uninfected variables or between humans and mosquitoes. When choosing each weight, we only take the limit that is critical for transmission of infection.

Now we know the overall Lyapunov function in the following form.[12]:

$V(X,Y) = U_1(X) + U_2(Y)$.

$$V(X,Y) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right) + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right) + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right) + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right) +$$
$$\alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right) + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right).$$

We are now analyzing the stability of the disease-free point using the Lyapunov function.

1- $$V(X,Y) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right) + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right) + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right) + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right) +$$
$$\alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right) + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right) > 0, \forall S_H, E_H, I_H, S_V, E_V, I_V \neq S_H^*, E_H^*, I_H^*, S_V^*, E_V^*, I_V^*.$$

2- $$V(X,Y) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right) + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right) + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right) + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right) +$$
$$\alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right) + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right) = 0, \forall\ S_H, S_V > 0, E_H, I_H, E_V, I_V = 0, \alpha_i > 0\ ; S_H, S_V = S_H^*, S_V^*.$$

After forming the Lyapunov function, we now move to its derivative with respect to time to study its change at the disease-free point.

3- $\dot{V}(X,Y) = \nabla U_1(X).F(X) + \nabla U_2(Y).F(Y)$.

$$\nabla U_1(X).F(X) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right)\frac{dS_H}{dt} + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right)\frac{dE_H}{dt} + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right)\frac{dI_H}{dt}.$$

$$\nabla U_2(Y).F(Y) = \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right)\frac{dS_V}{dt} + \alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right)\frac{dE_V}{dt} + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right)\frac{dI_V}{dt}.$$

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$\Rightarrow \dot{V}(X,Y) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right)\frac{dS_H}{dt} + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right)\frac{dE_H}{dt} + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right)\frac{dI_H}{dt} + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right)\frac{dS_V}{dt} + \alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right)\frac{dE_V}{dt} + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right)\frac{dI_V}{dt}.$

$\Rightarrow \dot{V}(X,Y) = \alpha_{S_H}\left(1 - \frac{S_H^*}{S_H}\right)\frac{dS_H}{dt} + \alpha_{E_H}\left(1 - \frac{E_H^*}{E_H}\right)\frac{dE_H}{dt} + \alpha_{I_H}\left(1 - \frac{I_H^*}{I_H}\right)\frac{dI_H}{dt} + \alpha_{S_V}\left(1 - \frac{S_V^*}{S_V}\right)\frac{dS_V}{dt} + \alpha_{E_V}\left(1 - \frac{E_V^*}{E_V}\right)\frac{dE_V}{dt} + \alpha_{I_V}\left(1 - \frac{I_V^*}{I_V}\right)\frac{dI_V}{dt}.$

We now substitute the values of the variables at the disease-free state, i.e. at the infection-free equilibrium point, into the derivative of the Lyapunov function $\left(\frac{\Lambda_H}{\mu_H}, 0,0,0, \frac{\Lambda_V}{\mu_V}, 0,0\right)$.

$\Rightarrow \dot{V}(X,Y) = (1)\left(1 - \frac{S_H^*}{S_H}\right)\frac{dS_H}{dt} + (1)\left(1 - \frac{(0)}{E_H}\right)\frac{dE_H}{dt} + \left(\frac{(\beta_H S_H I_H)}{(\alpha_1 E_H)}\right)\left(1 - \frac{(0)}{I_H}\right)\frac{dI_H}{dt} + \left(\frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}\right)\left(1 - \frac{S_V^*}{S_V}\right)\frac{dS_V}{dt} + \left(\frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}.\right)\left(1 - \frac{(0)}{E_V}\right)\frac{dE_V}{dt} + \left(\frac{(\beta_H S_H I_H)}{(\alpha_3 E_V)}\right)\left(1 - \frac{(0)}{I_V}\right)\frac{dI_V}{dt}.$

At DFE $S_H = S_H^*, S_V = S_V^*$ and $\frac{dE_H}{dt} = \frac{dI_H}{dt} = \frac{dE_V}{dt} = \frac{dI_V}{dt} = 0$.

$\Rightarrow \dot{V}(X,Y) = 0.$

Since we know the Lyapunov function of the system and have obtained its derivatives, we now resort to Lasalle's principle to determine whether the system at the point is radically stable or not.

We have shown that the function $V > 0$ for all points outside the disease-free point, that $V = 0$ at the disease-free point, and that its time derivative is$V \leq 0$. Then based on Lassalle's principle, we can conclude that the disease-free point is globally stable.

Now, we now turn to the study of the endemic point to evaluate the stability of the system in the presence of disease in the population.

Endemic Equilibrium (EE):

$\frac{dS_H}{dt} = 0 \Rightarrow \Lambda_H - \beta_H S_H I_H - \mu_H S_H + \rho R_H = 0 \Rightarrow S_H = \frac{(\Lambda_H + \rho R_H)}{(\beta_H I_H + \mu_H)}.$

$\frac{dE_H}{dt} = 0 \Rightarrow \beta_H S_H I_H - (\alpha_1 + \mu_H)E_H = 0 \Rightarrow E_H = \frac{\beta_H S_H I_H}{(\alpha_1 + \mu_H)}.$

$\frac{dI_H}{dt} = 0 \Rightarrow \alpha_1 E_H - (\alpha_2 + \mu_H + \delta)I_H + \psi I_H = 0 \Rightarrow I_H = \frac{\alpha_1 E_H}{(\alpha_2 + \mu_H + \delta - \psi)}.$

$\frac{dR_H}{dt} = 0 \Rightarrow \alpha_2 I_H - (\mu_H + \rho)R_H = 0 \Rightarrow R_H = \frac{\alpha_2 I_H}{(\mu_H + \rho)}.$

$\frac{dS_V}{dt} = 0 \Rightarrow \Lambda_V - \beta_V S_V I_V - \mu_V S_V = 0 \Rightarrow S_V = \frac{\Lambda_V}{(\beta_V I_V + \mu_V)}.$

$\frac{dE_V}{dt} = 0 \Rightarrow \beta_V S_V I_V - (\alpha_3 + \mu_V)E_V = 0 \Rightarrow E_V = \frac{\beta_V S_V I_V}{(\alpha_3 + \mu_V)}.$

$\frac{dI_V}{dt} = 0 \Rightarrow \alpha_3 E_V - \mu_V I_V = 0 \Rightarrow I_V = \frac{\alpha_3 E_V}{\mu_V}.$

Therefore $EE = \{S_H^*, E_H^*, I_H^*, R_H^*, S_V^*, E_V^*, I_V^*\}$.

$\left\{\frac{(\Lambda_H + \rho R_H^*)}{(\beta_H I_H^* + \mu_H)}, \frac{\beta_H S_H^* I_H^*}{(\alpha_1 + \mu_H)}, \frac{\alpha_1 E_H^*}{(\alpha_2 + \mu_H + \delta - \psi)}, \frac{\alpha_2 I_H^*}{(\mu_H + \rho)}, \frac{\Lambda_V}{(\beta_V I_V^* + \mu_V)}, \frac{\beta_V S_V^* I_V^*}{(\alpha_3 + \mu_V)}, \frac{\alpha_3 E_V^*}{\mu_V}\right\}.$

Stability analysis using the Lyapunov function of EE:

Similar to what was done in the case of the DFE point, we now apply the Lyapunov function to the endemic point to analyze the behavior of the system around it.

$V(X,Y) = U_1(X) + U_2(Y).$

$V(X,Y) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right) + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right) + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right) + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right) + \alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right) + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right).$

We are now analyzing the stability of the endemic point using the Lyapunov function.

1- $V(X,Y) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right) + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right) + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right) + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right) + \alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right) + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right) > 0, \forall S_H, E_H, I_H, S_V, E_V, I_V \neq S_H^*, E_H^*, I_H^*, S_V^*, E_V^*, I_V^*.$

2- $V(X,Y) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right) + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right) + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right) + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right) + \alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right) + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right) = 0, \forall S_H, E_H, I_H, S_V, E_V, I_V = S_H^*, E_H^*, I_H^*, R_H^*, S_V^*, E_V^*, I_V^*, \alpha_i > 0.$

We relied on the fact that each variable at the endemic point is equal to its reference value because at this point the system is in a state of equilibrium and the variables do not change with time.

After forming the Lyapunov function, we now move to its derivative with respect to time to study its change at the endimic point.

3- $\dot{V}(X,Y) = \nabla U_1(X).F(X) + \nabla U_2(Y).F(Y).$

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$$\Rightarrow \dot{V}(X,Y) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right)\frac{dS_H}{dt} + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right)\frac{dE_H}{dt} + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right)\frac{dI_H}{dt} + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right)\frac{dS_V}{dt} + \alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right)\frac{dE_V}{dt} + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right)\frac{dI_V}{dt}.$$

$$\Rightarrow \dot{V}(X,Y) = \alpha_{S_H}\left(1 - \frac{S_H^*}{S_H}\right)\frac{dS_H}{dt} + \alpha_{E_H}\left(1 - \frac{E_H^*}{E_H}\right)\frac{dE_H}{dt} + \alpha_{I_H}\left(1 - \frac{I_H^*}{I_H}\right)\frac{dI_H}{dt} + \alpha_{S_V}\left(1 - \frac{S_V^*}{S_V}\right)\frac{dS_V}{dt} + \alpha_{E_V}\left(1 - \frac{E_V^*}{E_V}\right)\frac{dE_V}{dt} + \alpha_{I_V}\left(1 - \frac{I_V^*}{I_V}\right)\frac{dI_V}{dt}.$$

We now substitute the values of the variables at the endemic state, , into the derivative of the Lyapunov function $\left(\frac{(\Lambda_H+\rho R_H^*)}{(\beta_H I_H^*+\mu_H)}, \frac{\beta_H S_H^* I_H^*}{(\alpha_1+\mu_H)}, \frac{\alpha_1 E_H^*}{(\alpha_2+\mu_H+\delta-\psi)}, \frac{\alpha_2 I_H^*}{(\mu_H+\rho)}, \frac{\Lambda_V}{(\beta_V I_V^*+\mu_V)}, \frac{\beta_V S_V^* I_V^*}{(\alpha_3+\mu_V)}, \frac{\alpha_3 E_V^*}{\mu_V}\right)$.

$$\Rightarrow \dot{V}(X,Y) = (1)\left(1 - \frac{\left(\frac{(\Lambda_H+\rho R_H^*)}{(\beta_H I_H^*+\mu_H)}\right)}{S_H}\right)\frac{dS_H}{dt} + (1)\left(1 - \frac{\left(\frac{\beta_H S_H^* I_H^*}{(\alpha_1+\mu_H)}\right)}{E_H}\right)\frac{dE_H}{dt} + \left(\frac{(\beta_H S_H I_H)}{(\alpha_1 E_H)}\right)\left(1 - \frac{\left(\frac{\alpha_1 E_H^*}{(\alpha_2+\mu_H+\delta-\psi)}\right)}{I_H}\right)\frac{dI_H}{dt} + \left(\frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}\right)\left(1 - \frac{\left(\frac{\Lambda_V}{(\beta_V I_V^*+\mu_V)}\right)}{S_V}\right)\frac{dS_V}{dt} + \left(\frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}.\right)\left(1 - \frac{\left(\frac{\beta_V S_V^* I_V^*}{(\alpha_3+\mu_V)}\right)}{E_V}\right)\frac{dE_V}{dt} + \left(\frac{(\beta_H S_H I_H)}{(\alpha_3 E_V)}\right)\left(1 - \frac{\left(\frac{\alpha_3 E_V^*}{\mu_V}\right)}{I_V}\right)\frac{dI_V}{dt}.$$

$$\Rightarrow \dot{V}(X,Y) = \left(1 - \left(\frac{(\Lambda_H+\rho R_H^*)}{S_H(\beta_H I_H^*+\mu_H)}\right)\right)(\Lambda_H - \beta_H S_H I_H - \mu_H S_H + \rho R_H\ ) + \left(1 - \left(\frac{\beta_H S_H^* I_H^*}{E_H(\alpha_1+\mu_H)}\right)\right)(\beta_H S_H I_H - (\alpha_1+\mu_H)E_H) + \left(\frac{(\beta_H S_H I_H)}{(\alpha_1 E_H)}\right)\left(1 - \left(\frac{\alpha_1 E_H^*}{I_H(\alpha_2+\mu_H+\delta-\psi)}\right)\right)(\alpha_1 E_H - (\alpha_2+\mu_H+\delta)I_H + \psi I_H) + \left(\frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}\right)\left(1 - \left(\frac{\Lambda_V}{S_V(\beta_V I_V^*+\mu_V)}\right)\right)(\Lambda_V - \beta_V S_V I_V - \mu_V S_V\ ) + \left(\frac{(\beta_H S_H I_H)}{(\beta_V S_V I_V)}.\right)\left(1 - \left(\frac{\beta_V S_V^* I_V^*}{E_V(\alpha_3+\mu_V)}\right)\right)(\beta_V S_V I_V - (\alpha_3+\mu_V)E_V) + \left(\frac{(\beta_H S_H I_H)}{(\alpha_3 E_V)}\right)\left(1 - \left(\frac{\alpha_3 E_V^*}{I_V \mu_V}\right)\right)(\alpha_3 E_V - \mu_V I_V).$$

When studying the endemic point, it is found that the derivative of the Lyapunov function takes a complex form, comprising both positive and negative terms, which makes it difficult to determine the sign of the derivative directly. Therefore, it is not possible to explicitly conclude whether the point is stable based solely on this derivative, and it is necessary to rely on other analytical methods or numerical studies to evaluate the system's stability.

Therefore, we now resort to the numerical interpretation of the malaria model using Euler's method, with the introduction of values for the variables at the endemic point, to study the behavior of the system at this point and evaluate its stability in the event of the presence of the disease among the population.

Numerical Representation

To support the results extracted from the mathematical analysis, a numerical simulation of the model was conducted via MATLAB, and the solution of the deterministic system is shown in the following figure:

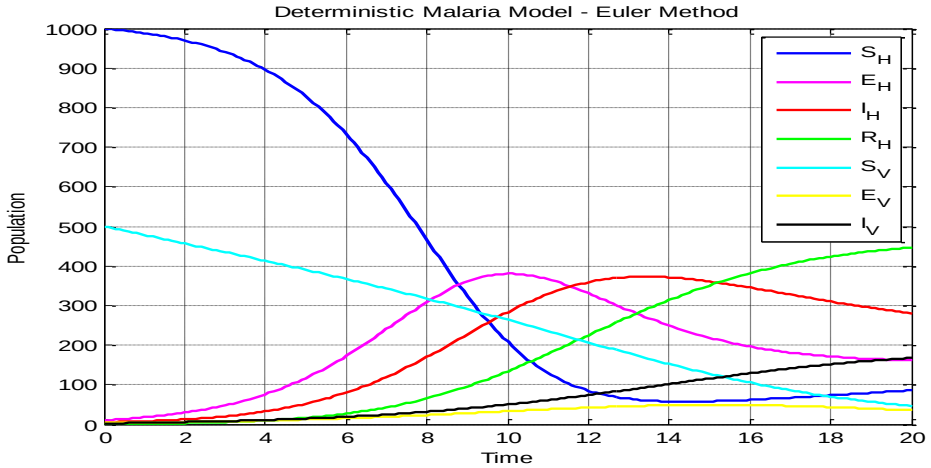

Figure 1. deterministic malaria model, $\Lambda_H = 10, \Lambda_V = 5, \beta_H = 0.002$, $\beta_V = 0.0015, \mu_H = 0.01, , \mu_V = 0.05, \rho = 0.1$, $\delta = 0.02, \psi = 0.01, \alpha_1 = 0.3, \alpha_2 = 0.2, \alpha_3 = 0.4$.

The numerical drawing of the deterministic model of malaria calculated by the Euler method shows the evolution of the number of human populations over time. We notice a decrease in the number of healthy people (S_H) and a gradual increase in the number of infected people (I_H) and recovered people (R_H), while the number of infected mosquitoes (I_V) increases and stabilizes after a period. This indicates that the system is approaching an endemic equilibrium.

Reflecting the continued spread of the disease in the community without disappearing, i.e., the DFE is unstable, and $R_0 > 1$.

After studying the numerical behavior of the deterministic system, we now move on to formulating the stochastic model of the system (1) to take the effect of randomness into account.

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The stochastic system model (1).[15]:

In biological and epidemiological systems, natural variables often undergo random changes as a result of environmental factors or individual fluctuations in society.

Therefore, it is necessary to convert the deterministic model (1) into a stochastic model to take these changes into account and analyze their impact on the disease dynamics. To represent these random effects, we assume the presence of Wiener-type noise, which allows us to simulate small, cumulative random changes in the system's variables, which better reflects the realistic nature of complex biological or physical processes.

Rewrite the model as

$$\begin{cases} \frac{dS_H}{dt} = -\beta_H S_H I_H - \mu_H S_H + \rho R_H \\ \frac{dE_H}{dt} = \beta_H S_H I_H - \alpha_1 E_H - \mu_H E_H \\ \frac{dI_H}{dt} = \alpha_1 E_H - \alpha_2 I_H - \mu_H I_H - \delta I_H + \psi I_H \\ \frac{dR_H}{dt} = \alpha_2 I_H - \mu_H R_H - \rho R_H \\ \frac{dS_V}{dt} = -\beta_V S_V I_V - \mu_V S_V \\ \frac{dE_V}{dt} = \beta_V S_V I_V - \alpha_3 E_V - \mu_V E_V \\ \frac{dI_V}{dt} = \alpha_3 E_V - \mu_V I_V \end{cases} \quad (2)$$

1- Probabilities associated with changes in the transmission of Malaria model:

Table 2. Probabilities associated with changes in the transmission of Malaria model

Changes,$\Delta \boldsymbol{x_i}$	Probability,$\boldsymbol{p_i}$
$(-1,1,0,0,0,0,0)^{\mathrm{tr}}$.	$\beta_H S_H I_H\ \Delta$t.
$(-1,0,0,0,0,0,0)^{\mathrm{tr}}$.	$\mu_H S_H\ \Delta t$.
$(1,0,0,-1,0,0,0)^{\mathrm{tr}}$.	$\rho R_H\ \Delta t$.
$(0,-1,1,0,0,0,0)^{\mathrm{tr}}$.	$\alpha_1 E_H\ \Delta t$.
$(0,-1,0,0,0,0,0)^{\mathrm{tr}}$.	$\mu_H E_H\ \Delta t$.
$(0,0,-1,1,0,0,0)^{\mathrm{tr}}$.	$\alpha_2 I_H\ \Delta t$.
$(0,0,-1,0,0,0,0)^{\mathrm{tr}}$.	$\mu_H I_H \Delta t$.
$(0,0,-1,0,0,0,0)^{\mathrm{tr}}$.	$\delta I_H\ \Delta t$.
$(0,0,1,0,0,0,0)^{\mathrm{tr}}$.	$\psi I_H\ \Delta t$.
$(0,0,0,-1,0,0,0)^{\mathrm{tr}}$.	$\mu_H R_H \Delta t$.
$(0,0,0,0,-1,1,0)^{\mathrm{tr}}$.	$\beta_V S_V I_V\ \Delta t$.
$(0,0,0,0,-1,0,0)^{\mathrm{tr}}$.	$\mu_V S_V\ \Delta t$.
$(0,0,0,0,0,-1,1)^{\mathrm{tr}}$.	$\alpha_3 E_V\ \Delta t$.
$(0,0,0,0,0,-1,0)^{\mathrm{tr}}$.	$\mu_V E_V\ \Delta t$.
$(0,0,0,0,0,0,-1)^{\mathrm{tr}}$.	$\mu_V I_V \Delta t$.

2- The expectation $E(\Delta x) = \sum_{i=1}^{15} p_i \Delta x_i$ is 7×1 matrix, the expectation can be expressed as follows.

$$E(\Delta x) = \sum_{i=1}^{15} p_i \Delta x_i = p_1 \Delta x_1 + p_2 \Delta x_2 + p_3 \Delta x_3 + \cdots + p_{15} \Delta x_{15}\ .$$

$$E(\Delta x) = \sum_{i=1}^{15} p_i \Delta x_i = \beta_H S_H I_H \begin{pmatrix} -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mu_H S_H \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \rho R_H \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \alpha_1 E_H \begin{pmatrix} 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mu_H E_H \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \alpha_2 I_H \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} +$$

$$\mu_H I_H \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \delta I_H \begin{pmatrix} 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \psi I_H \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \mu_H R_H \begin{pmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \\ 0 \end{pmatrix} +$$

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$$\beta_V S_V I_V \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 1 \\ 0 \end{pmatrix} + \mu_V S_V \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \alpha_3 E_V \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 1 \end{pmatrix} + \mu_V E_V \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \mu_V I_V \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{pmatrix}.$$

$$E(\Delta x) = \sum_{i=1}^{15} p_i \Delta x_i = \begin{pmatrix} -\beta_H S_H I_H \\ \beta_H S_H I_H \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\mu_H S_H \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} \rho R_H \\ 0 \\ 0 \\ -\rho R_H \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\alpha_1 E_H \\ \alpha_1 E_H \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ -\mu_H E_H \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\alpha_2 I_H \\ \alpha_2 I_H \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -\mu_H I_H \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 \\ 0 \\ -\delta I_H \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \psi I_H \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ -\mu_H R_H \\ 0 \\ 0 \\ 0 \end{pmatrix} +$$

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\beta_V S_V I_V \\ \beta_V S_V I_V \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\mu_V S_V \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\alpha_3 E_V \\ \alpha_3 E_V \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\mu_V E_V \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\mu_V I_V \end{pmatrix}.$$

$$E(\Delta x) = \begin{pmatrix} -\beta_H S_H I_H - \mu_H S_H + \rho R_H \\ \beta_H S_H I_H - \alpha_1 E_H - \mu_H E_H \\ \alpha_1 E_H - \alpha_2 I_H - \mu_H I_H - \delta I_H + \psi I_H \\ -\rho R_H + \alpha_2 I_H - \mu_H R_H \\ -\beta_V S_V I_V - \mu_V S_V \\ \beta_V S_V I_V - \alpha_3 E_V - \mu_V E_V \\ \alpha_3 E_V - \mu_V I_V \end{pmatrix} \Delta t.$$

3- The diffusion matrix G of dimension 7×15 is

$$G = \begin{pmatrix} -\sqrt{\beta_H S_H I_H} & -\sqrt{\mu_H S_H} & \sqrt{\rho R_H} & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\beta_H S_H I_H} & 0 & 0 & -\sqrt{\alpha_1 E_H} & -\sqrt{\mu_H E_H} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha_1 E_H} & 0 & -\sqrt{\alpha_2 I_H} & -\sqrt{\mu_H I_H} & -\sqrt{\delta I_H} \\ 0 & 0 & -\sqrt{\rho R_H} & 0 & 0 & \sqrt{\alpha_2 I_H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\psi I_H} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{\mu_H R_H} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\beta_V S_V I_V} & -\sqrt{\mu_V S_V} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\beta_V S_V I_V} & 0 & -\sqrt{\alpha_3 E_V} & -\sqrt{\mu_V E_V} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\alpha_3 E_V} & 0 & -\sqrt{\mu_V I_V} \end{pmatrix}.$$

4- we formulate the stochastic system as

$$dX(t) = f(X(t), t)dt + g(X(t), t)dW(t).$$

Where $dX(t) = \begin{bmatrix} dS_{H,t} \\ dE_{H,t} \\ dI_{H,t} \\ dR_{H,t} \\ dS_{V,t} \\ dE_{V,t} \\ dI_{V,t} \end{bmatrix}$, $f(X(t), t) = \left[\frac{E(\Delta X)}{\Delta t}\right]$, $g(X(t), t) = G$ and $dW(t) = \begin{bmatrix} dW_1(t) \\ dW_2(t) \\ \vdots \\ dW_{15}(t) \end{bmatrix}$.

Thus, the system takes the following form:

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$$\begin{pmatrix} dS_{H,t} \\ dE_{H,t} \\ dI_{H,t} \\ dR_{H,t} \\ dS_{V,t} \\ dE_{V,t} \\ dI_{V,t} \end{pmatrix} = \begin{pmatrix} -\beta_H S_H I_H - \mu_H S_H + \rho R_H \\ \beta_H S_H I_H - \alpha_1 E_H - \mu_H E_H \\ \alpha_1 E_H - \alpha_2 I_H - \mu_H I_H - \delta I_H + \psi I_H \\ -\rho R_H + \alpha_2 I_H - \mu_H R_H \\ -\beta_V S_V I_V - \mu_V S_V \\ \beta_V S_V I_V - \alpha_3 E_V - \mu_V E_V \\ \alpha_3 E_V - \mu_V I_V \end{pmatrix} dt +$$

$$\begin{pmatrix} -\sqrt{\beta_H S_H I_H} & -\sqrt{\mu_H S_H} & \sqrt{\rho R_H} & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\beta_H S_H I_H} & 0 & 0 & -\sqrt{\alpha_1 E_H} & -\sqrt{\mu_H E_H} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{\alpha_1 E_H} & 0 & -\sqrt{\alpha_2 I_H} & -\sqrt{\mu_H I_H} & -\sqrt{\delta I_H} \\ 0 & 0 & -\sqrt{\rho R_H} & 0 & 0 & \sqrt{\alpha_2 I_H} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \cdot$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\psi I_H} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\sqrt{\mu_H R_H} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{\beta_V S_V I_V} & -\sqrt{\mu_V S_V} & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\beta_V S_V I_V} & 0 & -\sqrt{\alpha_3 E_V} & -\sqrt{\mu_V E_V} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{\alpha_3 E_V} & 0 & -\sqrt{\mu_V I_V} \end{pmatrix} \cdot \begin{pmatrix} dW_1(t) \\ dW_2(t) \\ \cdot \\ \cdot \\ \cdot \\ dW_{15}(t) \end{pmatrix}.$$

$$\begin{cases} dS_{H,t} = (\rho R_H - \beta_H S_H I_H - \mu_H S_H)dt - \sqrt{\beta_H S_H I_H}\,dW_1(t) - \sqrt{\mu_H S_H}\,dW_2(t) + \sqrt{\rho R_H}\,dW_3(t) \\ dE_{H,t} = (\beta_H S_H I_H - \alpha_1 E_H - \mu_H E_H)dt + \sqrt{\beta_H S_H I_H}\,dW_1(t) - \sqrt{\alpha_1 E_H}\,dW_4(t) - \sqrt{\mu_H E_H}\,dW_5(t) \\ dI_{H,t} = (\alpha_1 E_H - \alpha_2 I_H - \mu_H I_H - \delta I_H + \psi I_H)dt + \sqrt{\alpha_1 E_H}\,dW_4(t) - \sqrt{\alpha_2 I_H}\,dW_6(t) - \sqrt{\mu_H I_H}\,dW_7(t) - \\ \qquad \sqrt{\delta I_H}\,dW_8(t) + \sqrt{\psi I_H}\,dW_9(t) \\ dR_{H,t} = (\alpha_2 I_H - \rho R_H - \mu_H R_H)dt - \sqrt{\rho R_H}\,dW_3(t) + \sqrt{\alpha_2 I_H}\,dW_6(t) - \sqrt{\mu_H R_H}\,dW_{10}(t) \\ dS_{V,t} = (-\beta_V S_V I_V - \mu_V S_V)dt - \sqrt{\beta_V S_V I_V}\,dW_{11}(t) - \sqrt{\mu_V S_V}\,dW_{12}(t) \\ dE_{V,t} = (\beta_V S_V I_V - \alpha_3 E_V - \mu_V E_V)dt + \sqrt{\beta_V S_V I_V}\,dW_{11}(t) - \sqrt{\alpha_3 E_V}\,dW_{13}(t) - \sqrt{\mu_V E_V}\,dW_{14}(t) \\ dI_t = (\alpha_3 E_V - \mu_V I_V)dt + \sqrt{\alpha_3 E_V}\,dW_{13}(t) - \sqrt{\mu_V I_V}\,dW_{15}(t) \end{cases} \quad (3)$$

After formulating the stochastic model of the system, we now move on to formulating the Lyapunov function to study the stability of the system under the influence of randomness.

$W(t) = (W_1(t), W_2(t), \dots, W_{15}(t))^T$ is independent standard wiener process $Z(t) = (S_H, E_H, I_H, R_H, S_V, E_V, I_V)^T$ is a seven-dimensional vector function, where T is transposition.

Introducing a lyapunov function:

$$V(t,Z) = \alpha_{S_H}\left(S_H - S_H^* - S_H^* ln\frac{S_H}{S_H^*}\right) + \alpha_{E_H}\left(E_H - E_H^* - E_H^* ln\frac{E_H}{E_H^*}\right) + \alpha_{I_H}\left(I_H - I_H^* - I_H^* ln\frac{I_H}{I_H^*}\right) + \alpha_{R_H}\left(R_H - R_H^* - R_H^* ln\frac{R_H}{R_H^*}\right) + \alpha_{S_V}\left(S_V - S_V^* - S_V^* ln\frac{S_V}{S_V^*}\right) + \alpha_{E_V}\left(E_V - E_V^* - E_V^* ln\frac{E_V}{E_V^*}\right) + \alpha_{I_V}\left(I_V - I_V^* - I_V^* ln\frac{I_V}{I_V^*}\right).$$

Applying the Ito lemma [16] :

$dV(t,Z) = LV(t,Z)dt + \sum_i^7 \frac{\partial V(t,Z)}{\partial x_i} g_i(t,Z)dW(t)$ where $dW(t) = \begin{bmatrix} dW_1(t) \\ dW_2(t) \\ \cdot \\ \cdot \\ \cdot \\ dW_{15}(t) \end{bmatrix}$.

Where $LV(t,Z) = \frac{\partial V}{\partial t}(t,Z) + \sum_{i=1}^6 f_i(t,Z)\frac{\partial V}{\partial x_i}(t,Z) + \frac{1}{2}\sum_{i,j}^6 [g(t,Z)g(t,Z)^T]_{ij}\frac{\partial^2 V}{\partial x_i \partial x_j}(t,Z)$.

$LV(t,Z) = \sum_{i=1}^7 f_i(t,Z)\frac{\partial V}{\partial x_i}(t,Z) + \frac{1}{2}\sum_{i,j}^7 [g(t,Z)g(t,Z)^T]_{ij}\frac{\partial^2 V}{\partial x_i \partial x_j}(t,Z)$.

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$$f_i(t,Z)\frac{\partial V}{\partial x_i}(t,Z)=\begin{pmatrix} f_1(t,Z)\frac{\partial V}{\partial x_1}(t,Z)\\ f_2(t,Z)\frac{\partial V}{\partial x_2}(t,Z)\\ f_3(t,Z)\frac{\partial V}{\partial x_3}(t,Z)\\ f_4(t,Z)\frac{\partial V}{\partial x_4}(t,Z)\\ f_5(t,Z)\frac{\partial V}{\partial x_5}(t,Z)\\ f_6(t,Z)\frac{\partial V}{\partial x_6}(t,Z)\\ f_7(t,Z)\frac{\partial V}{\partial x_7}(t,Z)\end{pmatrix}=\begin{pmatrix}(\Lambda_H-\beta_H S_H I_H-\mu_H S_H+\rho R_H\quad)\alpha_{S_H}\left(1-\frac{S_H^*}{S_H}\right)\\ (\beta_H S_H I_H-(\alpha_1+\mu_H)E_H\ )\ \alpha_{E_H}\left(1-\frac{E_H^*}{E_H}\right)\\ (\alpha_1 E_H-(\alpha_2+\mu_H+\delta)I_H+\psi I_H)\alpha_{I_H}\left(1-\frac{I_H^*}{I_H}\right)\\ (\alpha_2 I_H-(\mu_H+\rho)R_H\ )\quad\alpha_{R_V}\left(1-\frac{R_V^*}{R_V}\right)\\ (\Lambda_V-\beta_V S_V I_V-\mu_V S_V\quad)\ \alpha_{S_V}\left(1-\frac{S_V^*}{S_V}\right)\\ (\beta_V S_V I_V-(\alpha_3+\mu_V)E_V\quad)\alpha_{E_V}\left(1-\frac{E_V^*}{E_V}\right)\\ (\alpha_3 E_V-\mu_V I_V\quad)\alpha_{I_V}\left(1-\frac{I_V^*}{I_V}\right)\end{pmatrix},$$

$$\frac{1}{2}\sum_{i,j}^{7}[g(t,Z)g(t,Z)^T]_{ij}\frac{\partial^2 V}{\partial x_i\partial x_j}(t,Z)=\frac{1}{2}\Big(g_{1,1}\frac{\partial^2 V}{\partial x_1\partial x_1}+g_{1,2}\frac{\partial^2 V}{\partial x_1\partial x_2}+g_{1,4}\frac{\partial^2 V}{\partial x_1\partial x_4}+g_{2,1}\frac{\partial^2 V}{\partial x_2\partial x_1}+g_{2,2}\frac{\partial^2 V}{\partial x_2\partial x_2}+g_{2,3}\frac{\partial^2 V}{\partial x_2\partial x_3}+$$
$$g_{3,2}\frac{\partial^2 V}{\partial x_3\partial x_2}+g_{3,3}\frac{\partial^2 V}{\partial x_3\partial x_3}+g_{3,4}\frac{\partial^2 V}{\partial x_3\partial x_4}+g_{4,1}\frac{\partial^2 V}{\partial x_4\partial x_1}+g_{4,3}\frac{\partial^2 V}{\partial x_4\partial x_3}+g_{4,4}\frac{\partial^2 V}{\partial x_4\partial x_4}+g_{5,5}\frac{\partial^2 V}{\partial x_5\partial x_5}+g_{5,6}\frac{\partial^2 V}{\partial x_5\partial x_6}+g_{6,5}\frac{\partial^2 V}{\partial x_6\partial x_5}+$$
$$g_{6,6}\frac{\partial^2 V}{\partial x_6\partial x_6}+g_{6,7}\frac{\partial^2 V}{\partial x_6\partial x_7}+g_{7,6}\frac{\partial^2 V}{\partial x_7\partial x_6}+g_{7,7}\frac{\partial^2 V}{\partial x_7\partial x_7}\Big).$$

Where $g(t,Z)g(t,Z)^T$ is

$$\begin{pmatrix}\beta_H S_H I_H+\mu_H S_H+\rho R_H & -\beta_H S_H I_H & 0 & -\rho R_H & 0\\ -\beta_H S_H I_H & \beta_H S_H I_H+\alpha_1 E_H+\mu_H E_H & -\alpha_1 E_H & 0 & 0\\ 0 & -\alpha_1 E_H & \alpha_1 E_H+\alpha_2 I_H+\mu_H I_H+\delta I_H+\psi I_H & -\alpha_2 I_H & 0\\ \rho R_H & 0 & -\alpha_2 I_H & \rho R_H+\alpha_2 I_H+\mu_H R_H & 0\\ 0 & 0 & 0 & 0 & \beta_V S_V I_V+\mu_V S_V & -\beta_V\\ 0 & 0 & 0 & 0 & -\beta_V S_V I_V & \beta_V S_V I_V+\alpha\\ 0 & 0 & 0 & 0 & 0 & -\alpha\end{pmatrix}$$

.

$$LV(t,Z)=(\Lambda_H-\beta_H S_H I_H-\mu_H S_H+\rho R_H\quad)\alpha_{S_H}\left(1-\frac{S_H^*}{S_H}\right)+(\beta_H S_H I_H-(\alpha_1+\mu_H)E_H\ )\ \alpha_{E_H}\left(1-\frac{E_H^*}{E_H}\right)+$$
$$(\alpha_1 E_H-(\alpha_2+\mu_H+\delta)I_H+\psi I_H)\alpha_{I_H}\left(1-\frac{I_H^*}{I_H}\right)+(\alpha_2 I_H-(\mu_H+\rho)R_H\ )\ \alpha_{R_H}\left(1-\frac{R_H^*}{R_H}\right)+(\Lambda_V-\beta_V S_V I_V-\mu_V S_V\quad)\alpha_{S_V}\left(1-\frac{S_V^*}{S_V}\right)+(\beta_V S_V I_V-(\alpha_3+\mu_V)E_V\quad)\alpha_{E_V}\left(1-\frac{E_V^*}{E_V}\right)+(\alpha_3 E_V-\mu_V I_V\quad)\alpha_{I_V}\left(1-\frac{I_V^*}{I_V}\right)+\frac{1}{2}\Big((\beta_H S_H I_H+\mu_H S_H+\rho R_H)\frac{\partial^2 V}{\partial S_H\partial S_H}+$$
$$(-\beta_H S_H I_H)\frac{\partial^2 V}{\partial S_H\partial E_H}+(-\rho R_H)\frac{\partial^2 V}{\partial S_H\partial R_H}+(-\beta_H S_H I_H)\frac{\partial^2 V}{\partial E_H\partial S_H}+(\beta_H S_H I_H+\alpha_1 E_H+\mu_H E_H)\frac{\partial^2 V}{\partial E_H\partial E_H}+(-\alpha_1 E_H)\frac{\partial^2 V}{\partial E_H\partial I_H}+$$
$$(-\alpha_1 E_H)\frac{\partial^2 V}{\partial I_H\partial E_H}+(\alpha_1 E_H+\alpha_2 I_H+\mu_H I_H+\delta I_H+\psi I_H)\frac{\partial^2 V}{\partial I_H\partial I_H}+(-\alpha_2 I_H)\frac{\partial^2 V}{\partial I_H\partial R_H}+(\rho R_H)\frac{\partial^2 V}{\partial R_H\partial S_H}+(-\alpha_2 I_H)\frac{\partial^2 V}{\partial R_H\partial I_H}+$$
$$(\rho R_H+\alpha_2 I_H+\mu_H I_H)\frac{\partial^2 V}{\partial R_H\partial R_H}+(\beta_V S_V I_V+\mu_V S_V)\frac{\partial^2 V}{\partial S_V\partial S_V}+(-\beta_V S_V I_V)\frac{\partial^2 V}{\partial S_V\partial E_V}+(-\beta_V S_V I_V)\frac{\partial^2 V}{\partial E_V\partial S_V}+(\beta_V S_V I_V+\alpha_3 E_V+$$
$$\mu_V I_V)\frac{\partial^2 V}{\partial E_V\partial E_V}+(-\alpha_3 E_V)\frac{\partial^2 V}{\partial E_V\partial I_V}+(-\alpha_3 E_V)\frac{\partial^2 V}{\partial I_V\partial E_V}+(\alpha_3 E_V+\mu_V I_V)\frac{\partial^2 V}{\partial I_V\partial I_V}\Big).$$

$$LV(t,Z)=(\Lambda_H-\beta_H S_H I_H-\mu_H S_H+\rho R_H\quad)\alpha_{S_H}\left(1-\frac{S_H^*}{S_H}\right)+(\beta_H S_H I_H-(\alpha_1+\mu_H)E_H\ )\ \alpha_{E_H}\left(1-\frac{E_H^*}{E_H}\right)+$$
$$(\alpha_1 E_H-(\alpha_2+\mu_H+\delta)I_H+\psi I_H)\alpha_{I_H}\left(1-\frac{I_H^*}{I_H}\right)+(\alpha_2 I_H-(\mu_H+\rho)R_H\ )\ \alpha_{R_H}\left(1-\frac{R_H^*}{R_H}\right)+(\Lambda_V-\beta_V S_V I_V-\mu_V S_V\quad)\alpha_{S_V}\left(1-\frac{S_V^*}{S_V}\right)+(\beta_V S_V I_V-(\alpha_3+\mu_V)E_V\quad)\alpha_{E_V}\left(1-\frac{E_V^*}{E_V}\right)+(\alpha_3 E_V-\mu_V I_V\quad)\alpha_{I_V}\left(1-\frac{I_V^*}{I_V}\right)+\frac{1}{2}\Big((\beta_H S_H I_H+\mu_H S_H+\rho R_H)\frac{\partial^2 V}{\partial S_H\partial S_H}+$$
$$(-\beta_H S_H I_H)(0)+(-\rho R_H)(0)+(-\beta_H S_H I_H)(0)+(\beta_H S_H I_H+\alpha_1 E_H+\mu_H E_H)\frac{\partial^2 V}{\partial E_H\partial E_H}+(-\alpha_1 E_H)(0)+(-\alpha_1 E_H)(0)+$$
$$(\alpha_1 E_H+\alpha_2 I_H+\mu_H I_H+\delta I_H+\psi I_H)\frac{\partial^2 V}{\partial I_H\partial I_H}+(-\alpha_2 I_H)(0)+(\rho R_H)(0)+(-\alpha_2 I_H)(0)+(\rho R_H+\alpha_2 I_H+\mu_H I_H)\frac{\partial^2 V}{\partial R_H\partial R_H}+$$
$$(\beta_V S_V I_V+\mu_V S_V)\frac{\partial^2 V}{\partial S_V\partial S_V}+(-\beta_V S_V I_V)(0)+(-\beta_V S_V I_V)(0)+(\beta_V S_V I_V+\alpha_3 E_V+\mu_V I_V)\frac{\partial^2 V}{\partial E_V\partial E_V}+(-\alpha_3 E_V)(0)+(-\alpha_3 E_V)(0)+$$
$$(\alpha_3 E_V+\mu_V I_V)\frac{\partial^2 V}{\partial I_V\partial I_V}\Big).$$

$$LV(t,Z)=(\Lambda_H-\beta_H S_H I_H-\mu_H S_H+\rho R_H\quad)\alpha_{S_H}\left(1-\frac{S_H^*}{S_H}\right)+(\beta_H S_H I_H-(\alpha_1+\mu_H)E_H\ )\ \alpha_{E_H}\left(1-\frac{E_H^*}{E_H}\right)+$$
$$(\alpha_1 E_H-(\alpha_2+\mu_H+\delta)I_H+\psi I_H)\alpha_{I_H}\left(1-\frac{I_H^*}{I_H}\right)+(\alpha_2 I_H-(\mu_H+\rho)R_H\ )\ \alpha_{R_H}\left(1-\frac{R_H^*}{R_H}\right)+(\Lambda_V-\beta_V S_V I_V-\mu_V S_V\quad)\alpha_{S_V}\left(1-\frac{S_V^*}{S_V}\right)+(\beta_V S_V I_V-(\alpha_3+\mu_V)E_V\quad)\alpha_{E_V}\left(1-\frac{E_V^*}{E_V}\right)+(\alpha_3 E_V-\mu_V I_V\quad)\alpha_{I_V}\left(1-\frac{I_V^*}{I_V}\right)+\frac{1}{2}\Big((\beta_H S_H I_H+\mu_H S_H+\rho R_H)\frac{\partial^2 V}{\partial S_H\partial S_H}+$$

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$(\beta_H S_H I_H + \alpha_1 E_H + \mu_H E_H)\frac{\partial^2 V}{\partial E_H \partial E_H} + (\alpha_1 E_H + \alpha_2 I_H + \mu_H I_H + \delta I_H + \psi I_H)\frac{\partial^2 V}{\partial I_H \partial I_H} + (\rho R_H + \alpha_2 I_H + \mu_H I_H)\frac{\partial^2 V}{\partial R_H \partial R_H} +$
$(\beta_V S_V I_V + \mu_V S_V)\frac{\partial^2 V}{\partial S_V \partial S_V} + (\beta_V S_V I_V + \alpha_3 E_V + \mu_V I_V)\frac{\partial^2 V}{\partial E_V \partial E_V} + (\alpha_3 E_V + \mu_V I_V)\frac{\partial^2 V}{\partial I_V \partial I_V}\Big).$

Hence:

$dV(t,Z) = LV(t,Z)dt + \sum_i^7 \frac{\partial V(t,Z)}{\partial x_i} g_i(t,Z)dW(t)$ where $dW(t) = \begin{bmatrix} dW_1(t) \\ dW_2(t) \\ \cdot \\ \cdot \\ \cdot \\ dW_{15}(t) \end{bmatrix}$.

$$\sum_i^7 \frac{\partial V(t,Z)}{\partial x_i} g_i(t,Z)dW(t) = \sum_i^7 \frac{\partial V(t,Z)}{\partial x_i}.\begin{pmatrix} -\sqrt{\beta_H S_H I_H}dW_1(t) - \sqrt{\mu_H S_H}dW_2(t) + \sqrt{\rho R_H}\,dW_3(t) \\ \sqrt{\beta_H S_H I_H}dW_1(t) - \sqrt{\alpha_1 E_H}dW_4(t) - \sqrt{\mu_H E_H}dW_5(t) \\ \sqrt{\alpha_1 E_H}dW_4(t) - \sqrt{\alpha_2 I_H}dW_6(t) - \sqrt{\mu_H I_H}dW_7(t) - \\ \sqrt{\delta I_H}dW_8(t) + \sqrt{\psi I_H}dW_9(t) \\ \sqrt{\rho R_H}\,dW_3(t) + \sqrt{\alpha_2 I_H}dW_6(t) - \sqrt{\mu_H R_H}dW_{10}(t) \\ \sqrt{\beta_V S_V I_V}dW_{11}(t) - \sqrt{\mu_V S_V}dW_{12}(t) \\ \sqrt{\beta_V S_V I_V}dW_{11}(t) - \sqrt{\alpha_3 E_V}dW_{13}(t) - \sqrt{\mu_V E_V}dW_{14}(t) \\ \sqrt{\alpha_3 E_V}dW_{13}(t) - \sqrt{\mu_V I_V}dW_{15}(t) \end{pmatrix}.$$

$dV(t,Z) = \Big[(\Lambda_H - \beta_H S_H I_H - \mu_H S_H + \rho R_H\ )\alpha_{S_H}\left(1 - \frac{S_H^*}{S_H}\right) + (\beta_H S_H I_H - (\alpha_1 + \mu_H)E_H\ )\alpha_{E_H}\left(1 - \frac{E_H^*}{E_H}\right) +$
$(\alpha_1 E_H - (\alpha_2 + \mu_H + \delta)I_H + \psi I_H)\alpha_{I_H}\left(1 - \frac{I_H^*}{I_H}\right) + (\alpha_2 I_H - (\mu_H + \rho)R_H\ )\ \alpha_{R_V}\left(1 - \frac{R_V^*}{R_V}\right) + (\Lambda_V - \beta_V S_V I_V - \mu_V S_V\ )\alpha_{S_V}\left(1 -$
$\frac{S_V^*}{S_V}\right) + (\beta_V S_V I_V - (\alpha_3 + \mu_V)E_V\ )\alpha_{E_V}\left(1 - \frac{E_V^*}{E_V}\right) + (\alpha_3 E_V - \mu_V I_V\ )\alpha_{I_V}\left(1 - \frac{I_V^*}{I_V}\right) + \frac{1}{2}\Big((\beta_H S_H I_H + \mu_H S_H + \rho R_H)\frac{(\alpha_{S_H})^2 S_H^*}{(S_H)^2} +$
$(\beta_H S_H I_H + \alpha_1 E_H + \mu_H E_H)\frac{(\alpha_{E_H})^2 E_H^*}{(E_H)^2} + (\alpha_1 E_H + \alpha_2 I_H + \mu_H I_H + \delta I_H + \psi I_H)\frac{(\alpha_{I_H})^2 I_H^*}{(I_H)^2} + (\rho R_H + \alpha_2 I_H + \mu_H I_H)\frac{(\alpha_{R_H})^2 R_H^*}{(R_H)^2} +$
$(\beta_V S_V I_V + \mu_V S_V)\frac{(\alpha_{S_V})^2 S_V^*}{(S_V)^2} + (\beta_V S_V I_V + \alpha_3 E_V + \mu_V I_V)\frac{(\alpha_{E_V})^2 E_V^*}{(E_V)^2} + (\alpha_3 E_V + \mu_V I_V)\frac{(\alpha_{I_V})^2 I_V^*}{(I_V)^2}\Big)\Big]dt + \Big(-\sqrt{\beta_H S_H I_H}dW_1(t) -$
$\sqrt{\mu_H S_H}dW_2(t) + \sqrt{\rho R_H}\,dW_3(t)\Big)\frac{\partial V}{\partial S_H} + \Big(\sqrt{\beta_H S_H I_H}dW_1(t) - \sqrt{\alpha_1 E_H}dW_4(t) - \sqrt{\mu_H E_H}dW_5(t)\Big)\frac{\partial V}{\partial E_H} +$
$\Big(\sqrt{\alpha_1 E_H}dW_4(t) - \sqrt{\alpha_2 I_H}dW_6(t) - \sqrt{\mu_H I_H}dW_7(t) - \sqrt{\delta I_H}dW_8(t) + \sqrt{\psi I_H}dW_9(t)\Big)\frac{\partial V}{\partial I_H} + \Big(\sqrt{\rho R_H}\,dW_3(t) + \sqrt{\alpha_2 I_H}dW_6(t) -$
$\sqrt{\mu_H R_H}dW_{10}(t)\Big)\frac{\partial V}{\partial R_H} + \Big(\sqrt{\beta_V S_V I_V}dW_{11}(t) - \sqrt{\mu_V S_V}dW_{12}(t)\Big)\frac{\partial V}{\partial S_V} + \Big(\sqrt{\beta_V S_V I_V}dW_{11}(t) - \sqrt{\alpha_3 E_V}dW_{13}(t) -$
$\sqrt{\mu_V E_V}dW_{14}(t)\Big)\frac{\partial V}{\partial E_V} + \Big(\sqrt{\alpha_3 E_V}dW_{13}(t) - \sqrt{\mu_V I_V}dW_{15}(t)\Big)\frac{\partial V}{\partial I_V}.$

Then the Lyapunov function of stochastic Malaria model is

$V\big(t,Z(t)\big) = V(t_0,Z(t_0)) + \int_{t_0}^t LV(s,Z(s))ds + \int_{t_0}^t \sum_i^d \frac{\partial V(s,Z)}{\partial x_i} g_i(s,Z(s))dW(s).$

$V(t,Z) = V(t_0,Z(t_0)) + \int_{t_0}^t \Big[(\Lambda_H - \beta_H S_H I_H - \mu_H S_H + \rho R_H\ )\alpha_{S_H}\left(1 - \frac{S_H^*}{S_H}\right) + (\beta_H S_H I_H - (\alpha_1 + \mu_H)E_H\ )\alpha_{E_H}\left(1 - \frac{E_H^*}{E_H}\right) +$
$(\alpha_1 E_H - (\alpha_2 + \mu_H + \delta)I_H + \psi I_H)\alpha_{I_H}\left(1 - \frac{I_H^*}{I_H}\right) + (\alpha_2 I_H - (\mu_H + \rho)R_H\ )\ \alpha_{R_V}\left(1 - \frac{R_V^*}{R_V}\right) + (\Lambda_V - \beta_V S_V I_V - \mu_V S_V\ )\alpha_{S_V}\left(1 -$
$\frac{S_V^*}{S_V}\right) + (\beta_V S_V I_V - (\alpha_3 + \mu_V)E_V\ )\alpha_{E_V}\left(1 - \frac{E_V^*}{E_V}\right) + (\alpha_3 E_V - \mu_V I_V\ )\alpha_{I_V}\left(1 - \frac{I_V^*}{I_V}\right) + \frac{1}{2}\Big((\beta_H S_H I_H + \mu_H S_H + \rho R_H)\frac{(\alpha_{S_H})^2 S_H^*}{(S_H)^2} +$
$(\beta_H S_H I_H + \alpha_1 E_H + \mu_H E_H)\frac{(\alpha_{E_H})^2 E_H^*}{(E_H)^2} + (\alpha_1 E_H + \alpha_2 I_H + \mu_H I_H + \delta I_H + \psi I_H + \rho R_H)\frac{(\alpha_{I_H})^2 I_H^*}{(I_H)^2} + (\rho R_H + \mu_H I_H)\frac{(\alpha_{R_H})^2 R_H^*}{(R_H)^2} +$
$(\beta_V S_V I_V + \mu_V S_V)\frac{(\alpha_{S_V})^2 S_V^*}{(S_V)^2} + (\beta_V S_V I_V + \alpha_3 E_V + \mu_V I_V)\frac{(\alpha_{E_V})^2 E_V^*}{(E_V)^2} + (\alpha_3 E_V + \mu_V I_V)\frac{(\alpha_{I_V})^2 I_V^*}{(I_V)^2}\Big)\Big]ds + \int_{t_0}^t \Bigg(\Bigg(-\Bigg(\alpha_{S_H}\left(1 -$
$\frac{S_H^*}{S_H}\right)\Bigg)\sqrt{\beta_H S_H I_H}dW_1(s) - \left(\alpha_{S_H}\left(1 - \frac{S_H^*}{S_H}\right)\right)\sqrt{\mu_H S_H}dW_2(s) + (\alpha_{S_H}\left(1 - \frac{S_H^*}{S_H}\right))\sqrt{\rho R_H}\,dW_3(s)\Bigg) + \Bigg(\Bigg(\alpha_{E_H}\left(1 -$
$\frac{E_H^*}{E_H}\right)\Bigg)\sqrt{\beta_H S_H I_H}dW_1(s) - \left(\alpha_{E_H}\left(1 - \frac{E_H^*}{E_H}\right)\right)\sqrt{\alpha_1 E_H}dW_4(s) - (\alpha_{E_H}\left(1 - \frac{E_H^*}{E_H}\right))\sqrt{\mu_H E_H}dW_5(s)\Bigg) +$
$\Bigg((\alpha_{I_H}\left(1 - \frac{I_H^*}{I_H}\right))\sqrt{\alpha_1 E_H}dW_4(s) - (\alpha_{I_H}\left(1 - \frac{I_H^*}{I_H}\right))\sqrt{\alpha_2 I_H}dW_6(s) - (\alpha_{I_H}\left(1 - \frac{I_H^*}{I_H}\right))\sqrt{\mu_H I_H}dW_7(s) - (\alpha_{I_H}\left(1 - \frac{I_H^*}{I_H}\right))\sqrt{\delta I_H}dW_8(s) + (\alpha_{I_H}\left(1$

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$$\left(\left(\alpha_{R_V}\left(1-\frac{R_V^*}{R_V}\right)\right)\sqrt{\rho R_H}\,dW_3(s)+\left(\alpha_{R_V}\left(1-\frac{R_V^*}{R_V}\right)\right)\sqrt{\alpha_2 I_H}dW_6(s)-(\alpha_{R_V}\left(1-\frac{R_V^*}{R_V}\right))\sqrt{\mu_H R_H}dW_{10}(s)\right)+$$
$$\left(\left(\alpha_{S_V}\left(1-\frac{S_V^*}{S_V}\right)\right)\sqrt{\beta_V S_V I_V}dW_{11}(s)-(\alpha_{S_V}\left(1-\frac{S_V^*}{S_V}\right))\sqrt{\mu_V S_V}dW_{12}(s)\right)+\left(\left(\alpha_{E_V}\left(1-\frac{E_V^*}{E_V}\right)\right)\sqrt{\beta_V S_V I_V}dW_{11}(s)-\right.$$
$$\left.\left(\alpha_{E_V}\left(1-\frac{E_V^*}{E_V}\right)\right)\sqrt{\alpha_3 E_V}dW_{13}(s)-(\alpha_{E_V}\left(1-\frac{E_V^*}{E_V}\right))\sqrt{\mu_V E_V}dW_{14}(s)\right)+((\alpha_{I_V}\left(1-\frac{I_V^*}{I_V}\right))\sqrt{\alpha_3 E_V}dW_{13}(s)-(\alpha_{I_V}\left(1-\right.$$
$$\left.\left.\frac{I_V^*}{I_V}\right))\sqrt{\mu_V I_V}dW_{15}(s))\right).$$

And by taking the expectation of both sides we get

$$E[V(t,Z)]=V(t_0,Z(t_0))+E\left[\int_{t_0}^{t}\left[(\Lambda_H-\beta_H S_H I_H-\mu_H S_H+\rho R_H\ \ )\alpha_{S_H}\left(1-\frac{S_H^*}{S_H}\right)+(\beta_H S_H I_H-(\alpha_1+\mu_H)E_H\ )\alpha_{E_H}\left(1-\right.\right.\right.$$
$$\left.\frac{E_H^*}{E_H}\right)+(\alpha_1 E_H-(\alpha_2+\mu_H+\delta)I_H+\psi I_H)\alpha_{I_H}\left(1-\frac{I_H^*}{I_H}\right)+(\alpha_2 I_H-(\mu_H+\rho)R_H\ \ )\ \alpha_{R_V}\left(1-\frac{R_V^*}{R_V}\right)+(\Lambda_V-\beta_V S_V I_V-$$
$$\mu_V S_V\ \ )\alpha_{S_V}\left(1-\frac{S_V^*}{S_V}\right)+(\beta_V S_V I_V-(\alpha_3+\mu_V)E_V\ \ )\alpha_{E_V}\left(1-\frac{E_V^*}{E_V}\right)+(\alpha_3 E_V-\mu_V I_V\ \ )\alpha_{I_V}\left(1-\frac{I_V^*}{I_V}\right)+\frac{1}{2}\left((\beta_H S_H I_H+\mu_H S_H+\right.$$
$$\rho R_H)\frac{(\alpha_{S_H})^2 S_H^*}{(S_H)^2}+(\beta_H S_H I_H+\alpha_1 E_H+\mu_H E_H)\frac{(\alpha_{E_H})^2 E_H^*}{(E_H)^2}+(\alpha_1 E_H+\alpha_2 I_H+\mu_H I_H+\delta I_H+\psi I_H)\frac{(\alpha_{I_H})^2 I_H^*}{(I_H)^2}+(\rho R_H+\alpha_2 I_H+$$
$$\left.\left.\mu_H I_H)\frac{(\alpha_{R_H})^2 R_H^*}{(R_H)^2}+(\beta_V S_V I_V+\mu_V S_V)\frac{(\alpha_{S_V})^2 S_V^*}{(S_V)^2}+(\beta_V S_V I_V+\alpha_3 E_V+\mu_V I_V)\frac{(\alpha_{E_V})^2 E_V^*}{(E_V)^2}+(\alpha_3 E_V+\mu_V I_V)\frac{(\alpha_{I_V})^2 I_V^*}{(I_V)^2}\right)\right]ds\right].$$

To ensure the stability of the model in a random way using a Lyapunov function, the function V must always be positive, and its deterministic derivative must $LV \leq 0$ in the appropriate range. Under these conditions, it is V becomes a super-martingale., that is, its mathematical expectation does not exceed its initial value. When taking the mathematical expectation of both sides of the equation, the random part resulting from the integration disappears with respect to the Wiener process because this part represents martingale and expectation martingale equal to zero.[16]

Therefore, stability can be analyzed based on the deterministic part of the expression only. However, due to the complexity of the derivatives of the Lyapunov function and the difficulty of accurately calculating the mathematical expectation at a specific equilibrium point, this analysis was limited to studying the general properties of the function. It did not allow for a qualitative discussion of the system's behavior and stability in the average sense, based on the conditions that the proposed function fulfills.

Numerical Representation:

To support the results extracted from the mathematical analysis, a numerical simulation of the model was performed via MATLAB, and the solution to the random system is shown in the following figure:

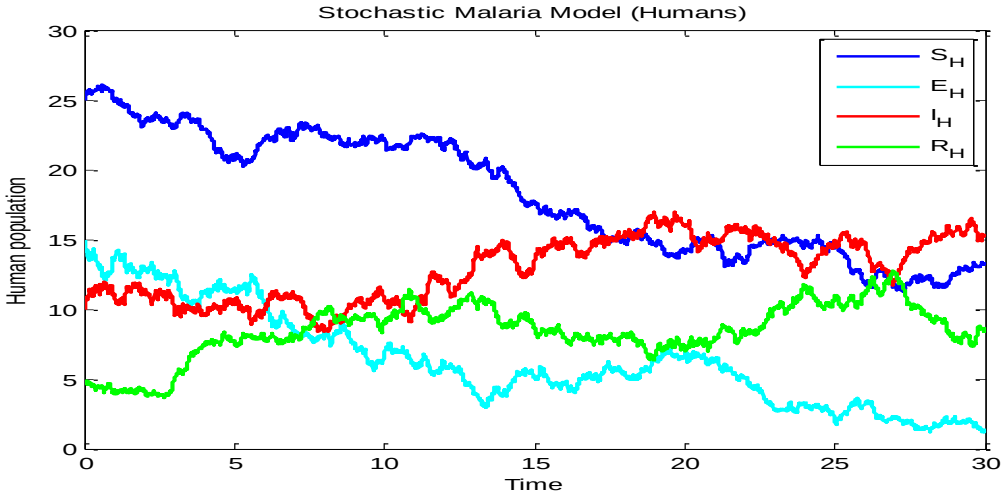

Figure 2. Stochastic malaria model (humans), $\Lambda_H = 10, \beta_H = 0.002$, $\mu_H = 0.01$, $\rho = 0.03$, $\delta = 0.005$, $\psi = 0.002, \alpha_1 = 0.1, \alpha_2 = 0.05$.

The figure illustrates the effect of randomness on the spread of malaria among humans. The number of healthy individuals (S_H) fluctuates sharply at the beginning, while the populations of susceptible (E_H) and infected (I_H) individuals change, reflecting the dynamics of the infection. The population of recovered individuals (R_H) gradually increases and decreases over time, due to loss of immunity or re-exposure.

In general, the diagram shows how different groups interact within a human society under the influence of randomness, which gives a realistic picture of the movement and spread of the disease.

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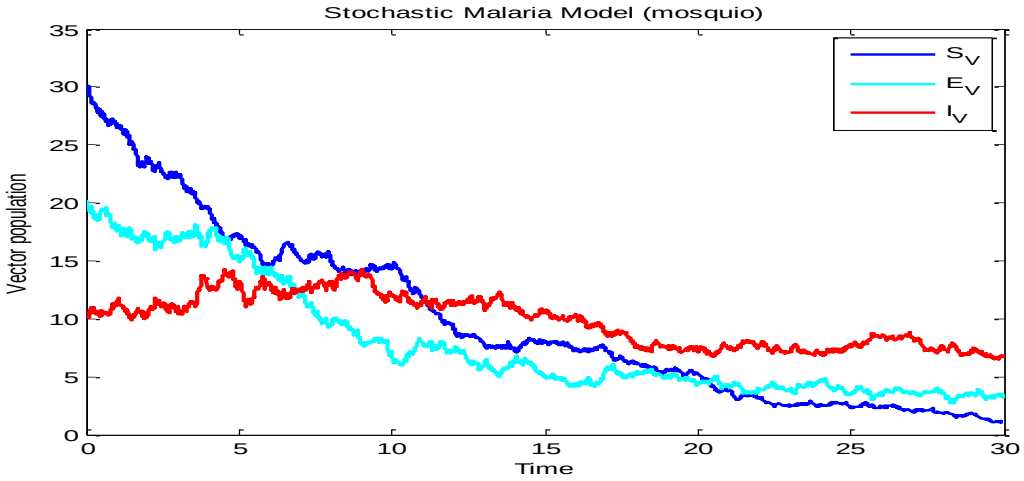

Figure 3. Stochastic malaria model (mosquito), $\Lambda_V = 5, \beta_V = 0.0015, \mu_V = 0.05, \alpha_3 = 0.08$.

The figure illustrates the fluctuations in mosquito populations in a stochastic malaria model. The number of healthy mosquitoes (S_V) appears to gradually decrease over time, while the numbers of susceptible (E_V) and infected (I_V) mosquitoes fluctuate continuously, reflecting the effect of randomness on each population during the period studied.

Results

This work presents the analytical and numerical findings derived from both the deterministic and stochastic formulations of the malaria transmission model. The deterministic analysis revealed two biologically meaningful equilibria: the disease-free equilibrium (E^0) and the endemic equilibrium (E^*).By applying the Next-Generation Matrix approach, the basic reproduction number (R_0) was expressed as the spectral radius of the matrix FM^{-1}.This threshold value governs the qualitative behavior of the system: when $R_0 < 1$, infection gradually disappears from the population; conversely, when $R_0 > 1$, malaria persists, and the system converges toward the endemic equilibrium. Using Lyapunov's direct method and LaSalle's Invariance Principle, the global stability of both equilibria was rigorously demonstrated.

A properly defined Lyapunov function $V(X)$ confirmed that the time derivative dV/dt is negative semi-definite within the feasible region, ensuring global convergence to E^0 or E^* depending on the value of R_0. This analytical result guarantees that the long-term system behavior is fully determined by the reproduction number, regardless of the initial state.To explore the effect of environmental variability, stochastic perturbations were introduced, transforming the deterministic system into a stochastic differential framework.

By employing Itô's formula on a stochastic Lyapunov functional $V(X,t)$, the extinction condition was obtained $R_0$where this condition indicates that random fluctuations can effectively lower the transmission threshold, promoting disease extinction even in scenarios where the deterministic model predicts persistence. The analytical findings were validated through numerical simulations using the Euler method for the deterministic system and the Euler–Maruyama method for the stochastic model. In all simulations, the trajectories of the deterministic system approached the predicted equilibria, while the stochastic trajectories demonstrated oscillatory damping that, for higher noise intensities, eventually led to extinction. These numerical outcomes are fully consistent with the theoretical expectations derived from the analytical framework.

Discussion

The comparative analysis of the deterministic and stochastic malaria models provides valuable insights into the interplay between system parameters and environmental fluctuations. In the deterministic scenario, malaria persistence is entirely dictated by the value of the basic reproduction number (R_0). However, the stochastic formulation introduces a new dimension of realism: environmental noise, often viewed as a destabilizing factor, can instead act as a stabilizing mechanism that drives the system toward extinction. This finding aligns with modern epidemiological understanding that random environmental variations, such as changes in temperature, humidity, or mosquito breeding conditions, can influence malaria transmission intensity.

Mathematically, the inclusion of white noise modifies the effective threshold for infection persistence, showing that stochasticity can suppress disease spread even in parameter regimes where the deterministic model predicts long-term endemicity. The significance of this result lies in its practical implications. It suggests that natural fluctuations and uncertainty may enhance disease control efforts rather than hinder them. By recognizing that random environmental effects can push the system below the effective reproduction threshold, public health strategies can better anticipate and exploit such stabilizing influences

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in designing sustainable intervention programs. Overall, the deterministic analysis offers theoretical clarity, while the stochastic model introduces environmental realism. Together, they provide a comprehensive perspective on malaria transmission dynamics that bridges mathematical rigor and biological relevance.

Conclusion

This study conducted an integrated analytical and numerical examination of malaria transmission under deterministic and stochastic frameworks. The basic reproduction number (R_0) was identified as the key threshold determining whether malaria persists or dies out. By employing Lyapunov stability theory and LaSalle's invariance principle, the global asymptotic stability of both the disease-free and endemic equilibria was rigorously demonstrated. Incorporating stochastic effects revealed that environmental noise can effectively reduce the reproduction threshold, leading to possible disease extinction even when deterministic models predict persistence. These results deepen the theoretical understanding of malaria dynamics and provide valuable insights for developing sustainable, evidence-based control strategies under environmental uncertainty.

Conflict of interest. Nil

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Appendix

1- Deterministic Malaria Model - Euler Method (MATLAB)

```
%% Deterministic Malaria Model - Euler Method
clear; clc; close all;
% PARAMETERS (example values)
Lambda_H = 10; Lambda_V = 5;
beta_H = 0.002; beta_V = 0.0015;
mu_H = 0.01; mu_V = 0.05;
rho = 0.1; delta = 0.02; psi = 0.01;
alpha_1 = 0.3; alpha_2 = 0.2; alpha_3 = 0.4;
% INITIAL CONDITIONS
S_H0 = 1000; E_H0 = 10; I_H0 = 5; R_H0 = 0;
S_V0 = 500;  E_V0 = 5;  I_V0 = 2;
y0 = [S_H0 E_H0 I_H0 R_H0 S_V0 E_V0 I_V0];
 % TIME PARAMETERS
T = 20;      % total time
dt = 0.1;     % time step
N = T/dt;     % number of steps
time = 0:dt:T;
% ALLOCATE ARRAYS
Y = zeros(N+1,7);
Y(1,:) = y0;
% EULER METHOD
for k = 1:N
    S_H = Y(k,1); E_H = Y(k,2); I_H = Y(k,3); R_H = Y(k,4);
```

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```
    S_V = Y(k,5); E_V = Y(k,6); I_V = Y(k,7);
     dS_H = Lambda_H - beta_H*S_H*I_H - mu_H*S_H + rho*R_H;
    dE_H = beta_H*S_H*I_H - (alpha_1 + mu_H)*E_H;
    dI_H = alpha_1*E_H - (alpha_2 + mu_H + delta)*I_H + psi*I_H;
    dR_H = alpha_2*I_H - (mu_H + rho)*R_H;
    dS_V = Lambda_V - beta_V*S_V*I_V - mu_V*S_V;
    dE_V = beta_V*S_V*I_V - (alpha_3 + mu_V)*E_V;
    dI_V = alpha_3*E_V - mu_V*I_V;
    Y(k+1,:) = Y(k,:) + dt*[dS_H dE_H dI_H dR_H dS_V dE_V dI_V];
end
% PLOT RESULTS
figure;
plot(time,Y(:,1),'b','LineWidth',1.5); hold on;
plot(time,Y(:,2),'m','LineWidth',1.5);
plot(time,Y(:,3),'r','LineWidth',1.5);
plot(time,Y(:,4),'g','LineWidth',1.5);
plot(time,Y(:,5),'c','LineWidth',1.5);
plot(time,Y(:,6),'y','LineWidth',1.5);
plot(time,Y(:,7),'k','LineWidth',1.5);
xlabel('Time'); ylabel('Population');
legend('S_H','E_H','I_H','R_H','S_V','E_V','I_V');
title('Deterministic Malaria Model - Euler Method');
grid on;
```

2- Stochastic Malaria Model - Euler-Maruyama

```
% PARAMETERS (Example values)
Lambda_H = 10;   % Not used in S_H since removed in SDE
beta_H = 0.002; alpha_1 = 0.1; alpha_2 = 0.05;
mu_H = 0.01; delta = 0.005; psi = 0.002; rho = 0.03;
beta_V = 0.0015; alpha_3 = 0.08; mu_V = 0.05;
% Simulation settings
T = 30;          % Total time
dt = 0.01;       % Time step
N = T/dt;        % Number of steps
% Initialize variables
S_H = zeros(1,N); E_H = zeros(1,N); I_H = zeros(1,N); R_H = zeros(1,N);
S_V = zeros(1,N); E_V = zeros(1,N); I_V = zeros(1,N);
% Initial conditions
S_H(1) = 25; E_H(1) = 15; I_H(1) = 10; R_H(1) = 5;
S_V(1) = 30; E_V(1) = 20; I_V(1) = 10;
% Simulation loop (Euler-Maruyama)
for i = 1:N-1
    dW = sqrt(dt)*randn(1,15); % Wiener increments
    % Human populations
    S_H(i+1) = S_H(i) + (rho*R_H(i) - beta_H*S_H(i)*I_H(i) - mu_H*S_H(i))*dt ...
            - sqrt(beta_H*S_H(i)*I_H(i))*dW(1) ...
            - sqrt(mu_H*S_H(i))*dW(2) ...
            + sqrt(rho*R_H(i))*dW(3);
    E_H(i+1) = E_H(i) + (beta_H*S_H(i)*I_H(i) - alpha_1*E_H(i) - mu_H*E_H(i))*dt ...
            + sqrt(beta_H*S_H(i)*I_H(i))*dW(1) ...
            - sqrt(alpha_1*E_H(i))*dW(4) ...
            - sqrt(mu_H*E_H(i))*dW(5);
    I_H(i+1) = I_H(i) + (alpha_1*E_H(i) - alpha_2*I_H(i) - mu_H*I_H(i) - delta*I_H(i) + psi*I_H(i))*dt ...
            + sqrt(alpha_1*E_H(i))*dW(4) ...
            - sqrt(alpha_2*I_H(i))*dW(6) ...
            - sqrt(mu_H*I_H(i))*dW(7) ...
            - sqrt(delta*I_H(i))*dW(8) ...
            + sqrt(psi*I_H(i))*dW(9);
    R_H(i+1) = R_H(i) + (alpha_2*I_H(i) - mu_H*R_H(i) - rho*R_H(i))*dt ...
            - sqrt(rho*R_H(i))*dW(3) ...
            + sqrt(alpha_2*I_H(i))*dW(6) ...
            - sqrt(mu_H*R_H(i))*dW(10);
    % Vector populations
    S_V(i+1) = S_V(i) + (- beta_V*S_V(i)*I_V(i) - mu_V*S_V(i))*dt ...
```

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```
            - sqrt(beta_V*S_V(i)*I_V(i))*dW(11) ...
            - sqrt(mu_V*S_V(i))*dW(12);
    E_V(i+1) = E_V(i) + (beta_V*S_V(i)*I_V(i) - alpha_3*E_V(i) - mu_V*E_V(i))*dt ...
            + sqrt(beta_V*S_V(i)*I_V(i))*dW(11) ...
            - sqrt(alpha_3*E_V(i))*dW(13) ...
            - sqrt(mu_V*E_V(i))*dW(14);
    I_V(i+1) = I_V(i) + (alpha_3*E_V(i) - mu_V*I_V(i))*dt ...
            + sqrt(alpha_3*E_V(i))*dW(13) ...
            - sqrt(mu_V*I_V(i))*dW(15);
    % Ensure non-negative populations
    S_H(i+1) = max(S_H(i+1),0); E_H(i+1) = max(E_H(i+1),0);
    I_H(i+1) = max(I_H(i+1),0); R_H(i+1) = max(R_H(i+1),0);
    S_V(i+1) = max(S_V(i+1),0); E_V(i+1) = max(E_V(i+1),0);
    I_V(i+1) = max(I_V(i+1),0);
end
% Plot results
time = 0:dt:T-dt;
figure;
plot(time,S_H,'b',time,E_H,'c',time,I_H,'r',time,R_H,'g','LineWidth',1.5);
xlabel('Time'); ylabel('Human population'); legend('S_H','E_H','I_H','R_H');
title('Stochastic Malaria Model (Humans)');
figure;
plot(time,S_V,'b',time,E_V,'c',time,I_V,'r','LineWidth',1.5);
xlabel('Time'); ylabel('Vector population'); legend('S_V','E_V','I_V');
title('Stochastic Malaria Model (mosquio)');
```

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